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Замечание о произведении двух формационных tcc-подгрупп[1]

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Аннотация

Подгруппа A группы G называется *tcc-подгруппой* в G, если существует подгруппа T группы G такая, что $G = AT$ и для любого $X \leqslant A$ и $Y \leqslant T$ существует элемент $u \in \langle X, Y \rangle$ такой, что $XY^u \leq G$. Запись $H \leqslant G$ означает, что H является подгруппой группы G. В этой статье мы исследуем группу $G = AB$ при условии, что A и B являются tcc-подгруппами в G. Доказано, что такая группа G принадлежит $\mathfrak{F}$, если подгруппы A и B принадлежат $\mathfrak{F}$, где $\mathfrak{F}$ — насыщенная формация такая, что $\mathfrak{U} \subseteq \mathfrak{F}$. Здесь $\mathfrak{U}$ — формация всех сверхразрешимых групп.

Ключевые слова: сверхразрешимая группа, тотально перестановочное произведение, насыщенная формация, tcc-перестановочное произведение, tcc-подгруппа.

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A remark on a product of two formational tcc-subgroups

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Abstract

A subgroup A of a group G is called *tcc-subgroup* in G, if there is a subgroup T of G such that $G = AT$ and for any $X \leqslant A$ and $Y \leqslant T$ there exists an element $u \in \langle X, Y \rangle$ such that $XY^u \leq G$. The notation $H \leqslant G$ means that H is a subgroup of a group G. In this paper we consider a group $G = AB$ such that A and B are tcc-subgroups in G. We prove that G belongs to $\mathfrak{F}$, when A and B belong to $\mathfrak{F}$ and $\mathfrak{F}$ is a saturated formation such that $\mathfrak{U} \subseteq \mathfrak{F}$. Here $\mathfrak{U}$ is the formation of all supersoluble groups.

Keywords: supersoluble group, totally permutable product, saturated formation, tcc-permutable product, tcc-subgroup.

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1. Introduction

Throughout this paper, all groups are finite and G always denotes a finite group. We use the standard notations and terminology of [1, 2]. The notation $H \leqslant G$ means that H is a subgroup of a group G.

It is well known that the product of two normal nilpotent subgroups of a group G is nilpotent. However, the product of two normal supersoluble subgroups of a group G is not necessarily supersoluble. It seems then natural to consider factorized groups in which certain subgroups of the corresponding factors permute, in order to obtain new criteria of supersolubility. A starting point of this research can be located at M. Asaad and A. Shaalan's paper [3]. In particular, they proved the supersolubility of a group $G = AB$ such that the subgroups A and B are totally permutable and supersoluble, see [3, Theorem 3.1]. Here the subgroups A and B of a group G are *totally permutable* if every subgroup of A is permutable with every subgroup of B. In [4] Maier showed that this statement is also true for the saturated formations containing the formation $\mathfrak{U}$ of all supersoluble groups. Ballester-Bolinches and Perez-Ramos in [5] extend Maier's result to non-saturated formations which contain all supersoluble groups. This direction have since been subject of an in-depth study of many authors, see, for example, [6], [7], [8]. The monograph [9, chapters 4–5] contains other detailed information on the structure of groups, which are totally or mutually permutable products of two subgroups.

The following concept was introduced in [8] .

Definition . *A subgroup A of a group G is called tcc-subgroup in G, if it satisfies the following conditions:*
1) *there is a subgroup T of G such that $G = AT$;*
2) *for any $X \leqslant A$ and $Y \leqslant T$ there exists an element $u \in \langle X, Y \rangle$ such that $XY^u \leq G$.*
We say that the subgroup T is a tcc-supplement to A in G.

Now, we can state the main result in [10], which is the following:

Theorem 1. ([10, Theorem A]) *Let $G = AB$, where A and B are tcc-subgroups in G. Let $\mathfrak{F}$ be a saturated formation of soluble groups such that $\mathfrak{U} \subseteq \mathfrak{F}$. Suppose that A and B belong to $\mathfrak{F}$. Then G belongs to $\mathfrak{F}$.*

In this article we show that the hypothesis of solubility in Theorem 1 can be removed.

Theorem 2. *Let $G = AB$, where A and B are tcc-subgroups in G. Let $\mathfrak{F}$ be a saturated formation such that $\mathfrak{U} \subseteq \mathfrak{F}$. Suppose that A and B belong to $\mathfrak{F}$. Then G belongs to $\mathfrak{F}$.*

2. Preliminaries

In this section, we give some definitions and basic results which are essential in the sequel.

A group whose chief factors have prime orders is called *supersoluble*. If $H \leqslant G$ and $H \neq G$, we write $H < G$. The notation $H \trianglelefteq G$ means that H is a normal subgroup of a group G. Denote by $Z(G)$, $F(G)$ and $\Phi(G)$ the centre, Fitting and Frattini subgroups of G respectively, and by $\mathrm{O}_p(G)$ the greatest normal p-subgroup of G. Denote by $\pi(G)$ the set of all prime divisors of order of G. The semidirect product of a normal subgroup A and a subgroup B is written as follows: $A \rtimes B$.

The monographs [11], [12] contain the necessary information of the theory of formations. A formation $\mathfrak{F}$ is said to be *saturated* if $G/\Phi(G) \in \mathfrak{F}$ implies $G \in \mathfrak{F}$. In view of Theorems 3.2 and 4.6 in [12, IV], for any non-empty saturated formation $\mathfrak{F}$ there exists a *formation function* f (that is, any function of the form $f : \mathbb{P} \to \{\text{formations}\}$) such that $\mathfrak{F} = LF(f) := \{G \mid G/F_p(G) \in f(p)$ for all primes p dividing $|G|\}$. Here $F_p(G) = O_{p',p}(G)$ is the greatest normal p-nilpotent subgroup of G [12, IV, Section 7]. Such a function is called a *local definition* of $\mathfrak{F}$. Moreover, in view of Proposition 5.4 in [12, III], every non-empty saturated formation $\mathfrak{F}$ has a unique local definition f (called the *canonical local definition* of $\mathfrak{F}$) such that $f(p) = \mathfrak{N}_p f(p) \subseteq \mathfrak{F}$ for all primes p, where $\mathfrak{N}_p f(p) = \emptyset$ if $f(p) = \emptyset$ and $\mathfrak{N}_p f(p)$ is the class of all groups A with $A^{f(p)} \leq O_p(A)$ whenever $f(p) \neq \emptyset$.

If H is a subgroup of G, then $H_G = \bigcap_{x \in G} H^x$ is called *the core* of H in G. If a group G contains a maximal subgroup M with trivial core, then G is said to be *primitive* and M is its *primitivator*. A simple check proves the following lemma.

Lemma 1. *Let $\mathfrak{F}$ be a saturated formation and G be a group. Assume that $G \notin \mathfrak{F}$, but $G/N \in \mathfrak{F}$ for all non-trivial normal subgroups N of G. Then G is a primitive group.*

Recall that the product $G = AB$ is said to be *tcc-permutable* [7], if for any $X \leqslant A$ and $Y \leqslant B$ there exists an element $u \in \langle X, Y \rangle$ such that $XY^u \leq G$. The subgroups A and B in this product are called *tcc-permutable*.

Lemma 2. ([7, Theorem 1, Proposition 1-2]) *Let $G = AB$ be the tcc-permutable product of subgroups A and B and N be a minimal normal subgroup of G. Then the following statements hold:*

(1) $\{A \cap N, B \cap N\} \subseteq \{1, N\}$;

(2) *if $N \leq A \cap B$ or $N \cap A = N \cap B = 1$, then $|N| = p$, where p is a prime.*

Lemma 3. ([13, Theorem 4]) *Let $G = AB$ be the tcc-permutable product of subgroups A and B. Then $[A, B] \leq F(G)$.*

Lemma 4. ([8, Lemma 3.1]) *Let A be a tcc-subgroup in G and Y be a tcc-supplement to A in G. Then the following statements hold:*

(1) *A is a tcc-subgroup in H for any subgroup H of G such that $A \leqslant H$;*

(2) *AN/N is a tcc-subgroup in G/N for any $N \trianglelefteq G$;*

(3) *for every $A_1 \trianglelefteq A$ and $X \leq Y$ there exists an element $y \in Y$ such that $A_1X^y \leq G$. In particular, $A_1M \leq G$ for some maximal subgroup M of Y and $A_1H \leq G$ for some Hall π-subgroup H of soluble Y and any $\pi \subseteq \pi(G)$;*

(4) *$A_1K \leq G$ for every subnormal subgroup K of Y and for every $A_1 \trianglelefteq A$;*

(5) *if $T \trianglelefteq G$ such that $T \leq A$ and $T \cap Y = 1$, then $T_1 \trianglelefteq G$ for every $T_1 \trianglelefteq A$ such that $T_1 \leq T$;*

(6) *if $T \trianglelefteq G$ such that $T \cap A = 1$ and $T \leq Y$, then $A_1 \leq N_G(T_1)$ for every $T_1 \trianglelefteq T$ and for every $A_1 \trianglelefteq A$.*

Lemma 5. *Let G be a group and N a unique minimal normal subgroup of G. If G has a proper tcc-subgroup A such that $A \neq 1$, then N is abelian.*

Proof. Since A is a tcc-subgroup, it follows that $G = AY$, A and Y are tcc-permutable. If $[A,Y]=1$, then $A \le C_G(Y)$. It is clear A and Y are normal in G. Thus $N \le A \cap Y$. By Lemma 2, $|N| = p$ and N is abelian. Therefore $[A,Y] \ne 1$ and $N \le [A,Y] \le F(G) \ne 1$ by Lemma 3. Hence N is abelian. □

Lemma 6. *Let $A \ne 1$ be a proper tcc-subgroup in a primitive group G and Y be a tcc-supplement to A in G. Suppose that N is a unique minimal normal subgroup of G. If $N \cap A = 1$ and $N \le Y$, then A is a cyclic group of order dividing $p-1$.*

Proof. Since $N \cap A = 1$ and $N \le Y$, by Lemma 4 (6), $A \le N_G(K)$ for any $K \trianglelefteq N$. By Lemma 5, N is an elementary abelian group. We fix an element $a \in A$. If $x \in N$, then $x^a \in \langle x\rangle$, since $A \le N_G(\langle x\rangle)$ by hypothesis. Hence $x^a = x^{m_x}$, where m_x is a positive integer and $1 \le m_x \le p$. If $y \in N \setminus \{x\}$, then

$$(xy)^a = (xy)^{m_{xy}} = x^{m_{xy}}y^{m_{xy}},\ (xy)^a = x^a y^a = x^{m_x}y^{m_y},$$

$$x^{m_{xy}}y^{m_{xy}} = x^{m_x}y^{m_y},\ x^{m_{xy}-m_x} = y^{m_y-m_{xy}} = 1,\ m_{xy} = m_x = m_y.$$

Therefore we can assume that $x^a = x^{n_a}$ for all $x \in N$, where $1 \le n_a \le p$ and n_a is a positive integer. Hence we have A induces a power automorphism group on N. By the Fundamental Homomorphism Theorem, $A/C_A(N)$ is isomorphic to a subgroup of $\mathrm{P}(N)$, where $\mathrm{P}(N)$ is the power automorphism group of N. Since N is abelian, it follows that $C_G(N) = N$ by [2, Theorem 4.41] and $C_A(N) = 1$. On the other hand, $\mathrm{P}(N)$ is a cyclic group of order $p-1$. Really $\mathrm{P}(N)$ is a group of scalar matrices over the field $\mathbf{P}$ consisting of p elements. Hence $\mathrm{P}(N)$ is isomorphic to the multiplicative group $\mathbf{P}^*$ of $\mathbf{P}$ and besides, $\mathbf{P}^*$ is a cyclic group of order $p-1$. Therefore A is a cyclic group of order dividing $p-1$. □

Lemma 7. *Let $\mathfrak{F}$ be a formation, G group, A and B subgroups of G such that A and B belong to $\mathfrak{F}$. If $[A,B]=1$, then $AB \in \mathfrak{F}$.*

Proof. Since

$$[A,B] = \langle [a,b] \mid a \in A,\ b \in B\rangle = 1,$$

it follows that $ab = ba$ for all $a \in A$, $b \in B$. Let

$$A \times B = \{(a,b) \mid a \in A,\ b \in B\},$$

$$(a_1,b_1)(a_2,b_2) = (a_1a_2, b_1b_2),\quad \forall a_1,a_2 \in A,\ b_1,b_2 \in B\ -$$

be the external direct product of groups A and B. Since $A \in \mathfrak{F}$, $B \in \mathfrak{F}$ and $\mathfrak{F}$ is a formation, we have $A \times B \in \mathfrak{F}$. Let $\varphi : A \times B \to AB$ be a function with $\varphi((a,b)) = ab$. It is clear that φ is a surjection. Because

$$\varphi((a_1,b_1)(a_2,b_2)) = \varphi((a_1a_2,b_1b_2)) = a_1a_2b_1b_2 =$$

$$= a_1b_1a_2b_2 = \varphi((a_1,b_1))\varphi((a_2,b_2)),$$

it follows that φ is an epimorphism. The core Ker φ contains all elements (a,b) such that $ab=1$. In this case $a = b^{-1} \in A \cap B \leqslant Z(G)$. By the Fundamental Homomorphism Theorem,

$$A \times B/\mathrm{Ker}\ \varphi \cong AB.$$

Since $A \times B \in \mathfrak{F}$ and $\mathfrak{F}$ is a formation, $A \times B/\mathrm{Ker}\ \varphi \in \mathfrak{F}$. Hence $AB \in \mathfrak{F}$. □

Lemma 8. ([14, Lemma 2.16]) *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$ and G be a group with a normal subgroup E such that $G/E \in \mathfrak{F}$. If E is cyclic, then $G \in \mathfrak{F}$.*

3. Proof of Theorem 2

Assume that the claim is false and let G be a minimal counterexample. Suppose that G is simple. By Lemma 3, A and B are normal in G, a contradiction. Hence let K be an arbitrary non-trivial normal subgroup of G. The quotients $AK/K \simeq A/A\cap K$ and $BK/K \simeq B/B\cap K$ are tcc-subgroups in G/K by Lemma 4 (2), $AK/K \simeq A/A\cap K \in \mathfrak{F}$ and $BK/K \simeq B/B\cap K \in \mathfrak{F}$, because $\mathfrak{F}$ is a formation. Hence the quotient $G/K = (AK/K)(BK/K) \in \mathfrak{F}$ by induction.

Since $\mathfrak{F}$ is a saturated formation, it follows that $\Phi(G) = 1$, G has a unique minimal normal subgroup N and G is primitive by Lemma 1. By Lemma 5, N is abelian and $F(G) = N = C_G(N) = \mathrm{O}_p(G)$, $G = N \rtimes M$, where $|N| = p^n$ and M is a primitivator.

By Lemma 2, is either $|N| = p$, or $N \leq A$ and $N \cap Y = 1$, or $N \cap A = 1$ and $N \leq Y$, where Y is a tcc-supplement to A in G. In the first case, by Lemma 8, $G \in \mathfrak{F}$. Suppose that $N \leq A$ and $N \cap Y = 1$. Since Y is a tcc-subgroup in G, it follows that by Lemma 6, Y is a cyclic group of order dividing $p-1$. Then $Y \in g(p)$, where g is the canonical local definition of $\mathfrak{U}$. Since $\mathfrak{U} \subseteq \mathfrak{F}$, we have by [12, Proposition IV.3.11], $g(p) \subseteq f(p)$, where f is the canonical local definition of $\mathfrak{F}$. Hence $Y \in f(p)$.

Let Q be a Sylow q-subgroup of Y. It is obvious that $Q \leq G_q$ for some Sylow subgroup G_q of G. Then we can always choose a primitivator H of G such that $Q \leq H$. Really $G_q = M_q^g$ and $G_q \leq M^g = H$ for some $g \in G$ and some Sylow q-subgroup M_q of M. It is clear that H is a maximal subgroup of G. If $N \leq H$, then $G = NM = NM^g = NH = H$, a contradiction. Hence $NH = G$. Because N is abelian, then $N \cap H = 1$ and H is a primitivator.

Since $A = A \cap G = A \cap NH = N(A \cap H)$, we have

$$G = AY = N(A \cap H)Y.$$

Prove that $(A \cap H)Y$ is a primitivator of G. Since

$$[A \cap H, Q] \leq [A, Y] = F(G) = N$$

by Lemma 3 and $[A \cap H, Q] \leq H$, it follows that $[A \cap H, Q] \leq H \cap N = 1$. Therefore $A \cap H \leq C_G(Q) = T$. Besides $Y \leq T$. Then

$$T = T \cap G = T \cap N(A \cap H)Y = (A \cap H)Y(N \cap T).$$

It is obvious that $N \cap T$ is normal in T and hence $N \cap T$ is normal in $G = N(A \cap H)Y = NT$, since N is abelian. Thus is either $N \leq T$, or $N \cap T = 1$. In the first case, $T = G$ and $Q \leq Z(G)$, a contradiction. Otherwise, $T = (A \cap H)Y$ and $G = N \rtimes T$. Hence $T = (A \cap H)Y$ is a primitivator of G. Thus we can always choose a primitivator M_1 of G such that $G = N \rtimes M_1$, $Y \leq M_1$ and $M_1 = (A \cap M_1)Y$.

Because $A \in \mathfrak{F}$, it follows that $A/F_p(A) \in f(p)$. Since $N = C_G(N)$ and $N \leq A$, we have that $N \leq F_p(A) = F(A)$. Let N_1 is a minimal normal subgroup of A such that $N_1 \leq N$. Then $F(A) \leq C_A(N_1)$ by [2, Lemma 4.21]. Since A is a tcc-subgroup in G, it follows that by Lemma 4 (5), N_1 is normal in G. Hence $N = N_1$ and $C_A(N_1) = C_A(N) = N$. Then $F_p(A) = N$ and $A \cap M_1 \simeq A/N \in f(p)$.

Since $f(p)$ is a formation, $A \cap M_1 \in f(p)$, $Y \in f(p)$ and $[A \cap M_1, Y] = 1$, it follows that $M_1 \in f(p)$ by Lemma 7. Because $N \in \mathfrak{N}_p$, we have $G \in \mathfrak{N}_p f(p) = f(p) \subseteq \mathfrak{F}$.

So, we assume that $N \cap A = 1$ and $N \leq Y$. Similarly, we can show that $N \cap B = 1$ and $N \leq X$, where X is a tcc-supplement to B in G. By Lemma 6, A and B are cyclic. Hence G is supersoluble and therefore $G \in \mathfrak{F}$. The theorem is proved.

4. Conclusion

Clear that by condition 2 of Definition 1, $G = AT$ is the tcc-permutable product of the subgroups A and T. If $G = AB$ is the tcc-permutable product of subgroups A and B, then the subgroups A and B are tcc-subgroups in G. The converse is false.

Example 1. *The dihedral group $G =< a > \rtimes < c >$, $|a| = 12$, $|c| = 2$ ([15], IdGroup=[24,6]) is the product of tcc-subgroups $A =< a^3c >$ of order 2 and $B = < a^{10} > \rtimes < c >$ of order 12. But A and B are not tcc-permutable. Indeed, there are the subgroups $X = A$ and $Y =< c >$ of A and B respectively such that doesn't exist $u \in \langle X, Y \rangle =< a^3 > \rtimes < c >$ such that $XY^u \leq G$.*

Hence we have the following result.

Corollary 1. 1. *Let A and B be tcc-subgroups in G and $G = AB$. If A and B are supersoluble, then G is supersoluble,* ([8, Theorem 4.1])

2. *Let $\mathfrak{F}$ be a saturated formation containing $\mathfrak{U}$. Let the group $G = HK$ be the tcc-permutable product of subgroups H and K. If $H \in \mathfrak{F}$ and $K \in \mathfrak{F}$, then $G \in \mathfrak{F}$,* ([13, Theorem 5]).

3. *Suppose that A and B are supersoluble subgroups of G and $G = AB$. Suppose further that A and B are totally permutable. Then G is supersoluble,* ([3, Theorem 3.1]).

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