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О дисперсивных по Оре $\mathfrak{F}$-гиперцентральных подгруппах конечных групп

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Аннотация

Рассматриваются только конечные группы. Пусть A — группа автоморфмизмов группы G, содержащая все внутренние автоморфизмы, и F — максимальный внутренний локальных экран насыщенной формации $\mathfrak{F}$. A-композиционный фактор H/K группы G называется A-$\mathfrak{F}$-центральным, если $A/C_A(H/K) \in F(p)$ для всех $p \in \pi(H/K)$. A-$\mathfrak{F}$-гиперцентром G называется наибольшая А-допустимая подгруппа G, все A-композиционные факторы ниже которой A-$\mathfrak{F}$-центральны. Обозначается $\mathrm{Z}_{\mathfrak{F}}(G, A)$.

Напомним, что группа G называется дисперсивной по Оре, если G имеет нормальную холлову $\{p_1, \dots, p_i\}$-подгруппу для $1 \leq i \leq n$, где $p_1 > \dots > p_n$ — все простые делители $|G|$. Главным результатом работы является: Пусть $\mathfrak{F}$ — наследственная насыщенная формация, F — её максимальный внутренний локальный экран и N — дисперсивная по Оре A-допустимая подгруппа группы G, где $\mathrm{Inn}G \leq A \leq \mathrm{Aut}G$. Тогда и только тогда $N \leq \mathrm{Z}_{\mathfrak{F}}(G, A)$, когда $N_A(P)/C_A(P) \in F(p)$ для любых силовской p-подгруппы P группы N и простого делителя p порядка N.

В качестве следствий были получены известные результаты Р. Бэра о нормальных подгруппах в сверхразрешимом гиперцентре и элементах гиперцентра.

Пусть G — группа. Напомним, что

$$L_n(G) = \{x \in G \mid [x, \alpha_1, \dots, \alpha_n] = 1 \ \forall \alpha_1, \dots, \alpha_n \in \mathrm{Aut}G\}$$

и G называется автонильпотентной, если $G = L_n(G)$ для некоторого натурального n. Из главного результата можно извлечь критерии автонильпотентности групп. В частности, группа G автонильпотентна тогда и только тогда, когда она является прямым произведением своих силовских подгрупп и группа автоморфизмов любой силовской p-подгруппы группы G является p-группой для любого простого делителя p порядка G. Приведены примеры автонильпотентных групп нечетного порядка.

Ключевые слова: Конечная группа, нильпотентная группа, сверхразрешимая группа, автонильпотентная группа, A-$\mathfrak{F}$-гиперцентр группы, наследственная насыщенная формация.

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On the $\mathfrak{F}$-hypercentral subgroups with the sylow tower property of finite groups

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Abstract

Throughout this paper all groups are finite. Let A be a group of automorphisms of a group G that contains all inner automorphisms of G and F be the canonical local definition of a saturated formation $\mathfrak{F}$. An A-composition factor H/K of G is called A-$\mathfrak{F}$-central if $A/C_A(H/K) \in F(p)$ for all $p \in \pi(H/K)$. The A-$\mathfrak{F}$-hypercenter of G is the largest A-admissible subgroup of G such that all its A-composition factors are A-$\mathfrak{F}$-central. Denoted by $\mathrm{Z}_{\mathfrak{F}}(G, A)$.

Recall that a group G satisfies the Sylow tower property if G has a normal Hall $\{p_1, \dots, p_i\}$-subgroup for all $1 \leq i \leq n$ where $p_1 > \dots > p_n$ are all prime divisors of $|G|$. The main result of this paper is: Let $\mathfrak{F}$ be a hereditary saturated formation, F be its canonical local definition and N be an A-admissible subgroup of a group G where $\mathrm{Inn}G \leq A \leq \mathrm{Aut}G$ that satisfies the Sylow tower property. Then $N \leq \mathrm{Z}_{\mathfrak{F}}(G, A)$ if and only if $N_A(P)/C_A(P) \in F(p)$ for all Sylow p-subgroups P of N and every prime divisor p of $|N|$.

As corollaries we obtained well known results of R. Baer about normal subgroups in the supersoluble hypercenter and elements in the hypercenter.

Let G be a group. Recall that $L_n(G) = \{x \in G \mid [x, \alpha_1, \dots, \alpha_n] = 1\ \forall \alpha_1, \dots, \alpha_n \in \mathrm{Aut}G\}$ and G is called autonilpotent if $G = L_n(G)$ for some natural n. The criteria of autonilpotency of a group also follow from the main result. In particular, a group G is autonilpotent if and only if it is the direct product of its Sylow subgroups and the automorphism group of a Sylow p-subgroup of G is a p-group for all prime divisors p of $|G|$. Examples of odd order autonilpotent groups were given.

Keywords: Finite group, nilpotent group, supersoluble group, autonilpotent group, A-$\mathfrak{F}$-hypercenter of a group, hereditary saturated formation.

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1. Introduction and results

Throughout this paper all groups are finite and G always denotes a finite group. Recall that $\mathrm{Aut}G$ and $\mathrm{Inn}G$ are the groups of all and inner automorphisms of G respectively.

Let A be a group of automorphisms of G. Kaloujnine [1] and Hall [2] showed that if A stabilizes some chain of subgroups of G, then A is nilpotent. Huppert [3] and Shemetkov [4] showed that if G has has A-admissible series with prime indexes, then A is supersoluble. Shemetkov [4] and Schmid [5] obtained analogues results for a solubly saturated formation $\mathfrak{F}$. Note that the $\mathfrak{F}$-hypercenter of G with respect to A played an important role in their research.

Recall that $\pi(G)$ is the set of all prime divisors of $|G|$. A *formation* is a class $\mathfrak{F}$ of groups with the following properties: (a) every homomorphic image of an $\mathfrak{F}$-group is an $\mathfrak{F}$-group, and (b) if G/M and G/N are $\mathfrak{F}$-groups, then $G/(M \cap N) \in \mathfrak{F}$. A formation $\mathfrak{F}$ is said to be: *saturated* if $G \in \mathfrak{F}$ whenever $G/\Phi(G) \in \mathfrak{F}$ where $\Phi(G)$ is the Frattini's subgroup of G; *hereditary* if $H \in \mathfrak{F}$ whenever $H \leq G \in \mathfrak{F}$. A function of the form $f : \mathbb{P} \to \{formations\}$ is called a *formation function*. Recall [6, p. 356] that a formation $\mathfrak{F}$ is called *local* if $\mathfrak{F} = (G \,|\, G/C_G(H/K) \in f(p)$ for every $p \in \pi(H/K)$ and every chief factor H/K of $G)$ for some formation function f. In this case f is called a *local definition* of $\mathfrak{F}$. By the Gaschütz-Lubeseder-Schmid theorem, a formation is local if and only if it is non-empty and saturated. Recall that if $\mathfrak{F}$ is a local formation, there exists a unique formation function F, defining $\mathfrak{F}$, such that $F(p) = \mathfrak{N}_pF(p) \subseteq \mathfrak{F}$ for every $p \in \mathbb{P}$ by Proposition 3.8 [6, p. 360]. In this case F is called the *canonical local definition* of $\mathfrak{F}$.

Let $\mathrm{Inn}G \leq A$ be a group of automorphisms of G and F be the canonical local definition of a local formation $\mathfrak{F}$. An A-composition factor H/K of G is called A-$\mathfrak{F}$-*central* if $A/C_A(H/K) \in F(p)$ for all $p \in \pi(H/K)$. The A-$\mathfrak{F}$-*hypercenter* of G is the largest A-admissible subgroup of G such that all its A-composition factors are A-$\mathfrak{F}$-central. This subgroup always exists by Lemma 6.4 [6, p. 387]. It is denoted by $\mathrm{Z}_{\mathfrak{F}}(G, A)$. If $A = \mathrm{Inn}G$, then it is just the $\mathfrak{F}$-hypercenter $\mathrm{Z}_{\mathfrak{F}}(G)$ of G. If $\mathfrak{F} = \mathfrak{N}$ is the class of all nilpotent groups, then we use $\mathrm{Z}_\infty(G, A)$ to denote the A-hypercenter $\mathrm{Z}_{\mathfrak{N}}(G, A)$ of G. Recently the subgroups of $\mathrm{Z}_\infty(G, A)$ have been studied for example in [7, 8, 9, 10, 11].

Recall that Syl_pG is the set of all Sylow subgroups of G; G *satisfies the Sylow tower property* if G has a normal Hall $\{p_1, \dots, p_i\}$-subgroup for all $1 \leq i \leq n$ where $p_1 > \dots > p_n$ are all prime divisors of $|G|$. It is well known that a supersoluble group satisfies the Sylow tower property. Recently series of hereditary saturated formations of groups that satisfy the Sylow tower property have been constructed (see, [12, 13, 14, 15]).

THEOREM 1. *Let $\mathfrak{F}$ be a hereditary saturated formation, F be its canonical local definition and N be an A-admissible subgroup of G where* $\mathrm{Inn}G \leq A \leq \mathrm{Aut}G$ *that satisfies the Sylow tower property. Then $N \leq \mathrm{Z}_{\mathfrak{F}}(G, A)$ if and only if $N_A(P)/C_A(P) \in F(p)$ for all $P \in \mathrm{Syl}_p(N)$ and $p \in \pi(N)$.*

Author obtained particular cases of this theorem for $A = \mathrm{Inn}G$ and two formations of supersoluble type in [16, 17]. Recall that a group G is called strictly p-closed if $G/\mathrm{O}_p(G)$ is abelian of exponent dividing $p - 1$. We use $\mathfrak{U}$ to denote the class of all supersoluble groups.

COROLLARY 1 (R. Baer [18]). *Let N be a normal subgroup of G. Then $N \leq \mathrm{Z}_{\mathfrak{U}}(G)$ if and only if N satisfies the Sylow tower property and $N_G(P)/C_G(P)$ is strictly p-closed for all $P \in \mathrm{Syl}_p(N)$ and $p \in \pi(N)$.*

M. R. R. Moghaddam and M. A. Rostamyari (see [9]) introduced the concept of autonilpotent group. Let $L_n(G) = \{x \in G \mid [x, \alpha_1, \dots, \alpha_n] = 1 \ \forall \alpha_1, \dots, \alpha_n \in \mathrm{Aut}G\}$. Then G is called *autonilpotent* if $G = L_n(G)$ for some natural n. Some properties of autonilpotent groups were studied in [9]. In [8] all abelian autonilpotent groups were described. In particular, abelian autonilpotent non-unit groups of odd order don't exist. It was shown that if a p-group G is autonilpotent, then $\mathrm{Aut}G$ is a p-group (Theorem 2.2 [10]). In [11, p. 45] it was asked: "Does there exist any odd order autonilpotent group?"

COROLLARY 2. *Let p be a prime. A p-group G is autonilpotent if and only if* $\mathrm{Aut}G$ *is a p-group.*

An example of a p-group G of order p^5 $(p > 3)$ such that $\mathrm{Aut}G$ is also a p-group was constructed in [19]. In the library of small groups of GAP [20] there are 30 groups of order 3^6 such that their automorphism groups are also 3-groups (for example groups [729, 31], [729, 41] and [729, 46]). Hence the answer on the question from [11] is positive. From Theorem 2.3 [9] and Lemma 2.9 [10] it follows that a group is autonilpotent iff it is the direct product of its autonilpotent Sylow subgroups.

Corollary 3. *A group G is autonilpotent if and only if it is the direct product of its Sylow subgroups and the automorphism group of a Sylow p-subgroup of G is a p-group for all $p \in \pi(G)$.*

The proof of Corollary 3 doesn't use results from [9, 10]. The following result gives the description of elements in $\mathrm{Z}_\infty(G, A)$.

Corollary 4. *Let g be a p-element of a group G and $\mathrm{Inn}G \leq A \leq \mathrm{Aut}G$. Then $g \in \mathrm{Z}_\infty(G, A)$ if and only if $g^\alpha = g$ for every p'-element α of A.*

Corollary 5 (R. Baer [21]). *Let p be a prime and G be a group. Then a p-element g of G belongs to $\mathrm{Z}_\infty(G)$ if and only if it permutes with all p'-elements of G.*

Corollary 6. *A group G is autonilpotent if and only if every automorphism α of G fixes all elements of G whose orders are coprime to the order of α.*

According to Frobenius p-nilpotency criterion (see Theorem 5.26 [22, p. 171]) a group G is nilpotent if and only if $N_G(P)/C_G(P)$ is a p-group for every p-subgroup P of G and every $p \in \pi(G)$.

Corollary 7. *A group G is autonilpotent if and only if $N_{\mathrm{Aut}G}(P)/C_{\mathrm{Aut}G}(P)$ is a p-group for every p-subgroup P of G and every $p \in \pi(G)$.*

2. Proves of the results

Lemma 1 (Lemma 3.6 [16]). *Let P be a p-subgroup of a group G and R be a normal r-subgroup of G where $r \neq p$ are primes. Then $N_G(P)R/R = N_{G/R}(PR/R)$, $C_G(P)R/R = C_{G/R}(PR/R)$ and $N_G(P)/C_G(P) \simeq N_{G/R}(PR/R)/C_{G/R}(PR/R)$.*

Proof. [*Proof of Theorem* 1] Let prove Theorem 1 for $A = \mathrm{Inn}G$. In this case $\mathrm{Z}_\mathfrak{F}(G, A) = \mathrm{Z}_\mathfrak{F}(G)$.

Sufficiency. Let N be a normal subgroup of $\mathrm{Z}_\mathfrak{F}(G)$ with the Sylow tower property. So N has a normal Sylow q-subgroup Q. Note that $Q \trianglelefteq G$ and $Q \leq \mathrm{Z}_\mathfrak{F}(G)$.

Hence $G/C_G(Q) = N_G(Q)/C_G(Q) \in F(q)$ by Lemma 2.5 from [23]. If $Q = N$, then sufficiency is proven.

Let $Q < N$. From $N \leq \mathrm{Z}_\mathfrak{F}(G)$ it follows that $N/Q \leq \mathrm{Z}_\mathfrak{F}(G/Q)$. Using induction on the order of G, we may assume that $N_{G/Q}(PQ/Q)/C_{G/Q}(PQ/Q) \in F(p)$ for every $P \in \mathrm{Syl}_p N$ and $p \in \pi(N) \setminus \{q\}$. Hence $N_G(P)/C_G(P) \simeq N_{G/Q}(PQ/Q)/C_{G/Q}(PQ/Q) \in F(p)$ by Lemma 1. Thus sufficiency is proved.

Necessity. Let a group G be a minimal order counterexample with a normal subgroup $N \not\leq \mathrm{Z}_\mathfrak{F}(G)$ that satisfies the statement of Theorem 1 and p be the greatest prime divisor of $|N|$. Then a Sylow p-subgroup P of N is normal in G. Let H/K be a chief factor of G and $H \leq P$. Since $C_G(P) \leq C_G(H/K)$ and $G/C_G(P) = N_G(P)/C_G(P) \in F(p)$, $G/C_G(H/K) \in F(p)$. It means that $P \leq \mathrm{Z}_\mathfrak{F}(G)$. Note that $N_{G/P}(RP/P)/C_{G/P}(RP/P) \simeq N_G(R)/C_G(R) \in F(p)$ for every $R \in \mathrm{Syl}_r N$ and $r \in \pi(N) \setminus \{p\}$ by Lemma 1. From $|G/P| < |G|$ it follows that $N/P \leq \mathrm{Z}_\mathfrak{F}(G/P)$. Thus $N \leq \mathrm{Z}_\mathfrak{F}(G)$, the contradiction.

Assume now that $\mathrm{Inn}G \leq A \leq \mathrm{Aut}G$. Let $\Gamma = G \rtimes A$. From $\mathrm{Inn}G \leq A$ it follows that groups of automorphisms that are induced by A and Γ on a given section of G are isomorphic. It means that $N_A(H/K)/C_A(H/K) \simeq N_\Gamma(H/K)/C_\Gamma(H/K)$ for a given section H/K of G. In particular, every A-composition A-$\mathfrak{F}$-central factor of G is an $\mathfrak{F}$-central chief factor of Γ.

It means that A-admissible subgroup N of G that satisfies the Sylow tower property lies in $\mathrm{Z}_\mathfrak{F}(G, A)$ if and only if $N \leq \mathrm{Z}_\mathfrak{F}(\Gamma)$. The later is equivalent to $N_\Gamma(P)/C_\Gamma(P) \simeq N_A(P)/C_A(P) \in F(p)$ for every $P \in \mathrm{Syl}_p N$ and $p \in \pi(N)$. □

Proof. [*Proof of Corollary* 1] Recall that $\mathfrak{U}$ has the canonical local definition F where $F(p)$ is the class of all strictly p-closed groups, every subgroup of $\mathrm{Z}_\mathfrak{U}(G)$ is supersoluble and every supersoluble group satisfies the Sylow tower property. Thus Corollary 1 directly follows from Theorem 1. □

PROPOSITION 3. *A group G is autonilpotent if and only if $G = \mathrm{Z}_\infty(G, \mathrm{Aut}G)$.*

PROOF. [*Proof*] Recall that the canonical local definition of $\mathfrak{N}$ is $F(p) = \mathfrak{N}_p$ where $\mathfrak{N}_p$ is the class of all p-groups. Note that $\mathrm{Aut}G/C_{\mathrm{Aut}G}(L_i(G)/L_{i-1}(G)) \simeq 1 \in F(p)$ for all prime p. Hence every $\mathrm{Aut}G$-composition factor of G between $L_{i-1}(G)$ and $L_i(G)$ is $\mathrm{Aut}G$-$\mathfrak{N}$-central. It means that $L_i(G) \leq \mathrm{Z}_\infty(G, \mathrm{Aut}G)$ for every i. So if G is autonilpotent group, then $G = \mathrm{Z}_\infty(G, \mathrm{Aut}G)$.

Assume that $G = \mathrm{Z}_\infty(G, \mathrm{Aut}G)$. Hence there exists an $\mathrm{Aut}G$-composition series

$$1 = G_0 \leq \cdots \leq G_n = G$$

with $\mathrm{Aut}G$-$\mathfrak{N}$-central chief factors. Let $\Gamma = G \rtimes \mathrm{Aut}G$. Note that

$$\mathrm{Aut}G/C_{\mathrm{Aut}G}(G_i/G_{i-1}) \simeq \Gamma/C_\Gamma(G_i/G_{i-1}) \in \mathfrak{N}_p$$

for every $p \in \pi(G_i/G_{i-1})$ by analogy with the proof of Theorem 1. Note that $\Gamma/C_\Gamma(G_i/G_{i-1})$ does not have non-trivial p-subgroups for all $p \in \pi(G_i/G_{i-1})$ by Lemma 3.9 [24, p. 26]. So $\mathrm{Aut}G = C_{\mathrm{Aut}G}(G_i/G_{i-1})$. Hence $[G_i, \mathrm{Aut}G] \leq G_{i-1}$. It means that $[x, \alpha_1, \ldots, \alpha_n] = 1$ for all $x \in G$ and $\alpha_1, \ldots, \alpha_n \in \mathrm{Aut}G$. Thus $G = L_n(G)$ and G is autonilpotent. □

PROOF. [*Proof of Corollary* 3] Note that every nilpotent group satisfies the Sylow tower property and every autonilpotent group is nilpotent.

So a nilpotent group is autonilpotent if and only if $N_{\mathrm{Aut}G}(P)/C_{\mathrm{Aut}G}(P) \in \mathfrak{N}_p$ for every $P \in \mathrm{Syl}_p G$ and $p \in \pi(G)$ by Proposition 3 and Theorem 1. The automorphism group of a direct product of groups was described in [25]. In particular, if $G = P \times H$, where P is a Sylow subgroup of G, then $\mathrm{Aut}G = \mathrm{Aut}P \times \mathrm{Aut}H$ and $N_{\mathrm{Aut}G}(P)/C_{\mathrm{Aut}G}(P) \simeq \mathrm{Aut}P$. □

PROOF. [*Proof of Corollary* 2] Directly follows from Corollary 3. □

PROOF. [*Proof of Corollary* 4] From $\mathrm{Inn}G \leq A$ it follows that $\mathrm{Z}_\infty(G, A) \leq \mathrm{Z}_\infty(G)$ is nilpotent. Hence every Sylow subgroup of $\mathrm{Z}_\infty(G, A)$ is A-admissible. So

$$N_A(P)/C_A(P) = A/C_A(P) \in \mathfrak{N}_p$$

for every $P \in \mathrm{Syl}_p \mathrm{Z}_\infty(G, A)$ and $p \in \pi(\mathrm{Z}_\infty(G, A))$ by Theorem 1. It means that if g is a p-element of $\mathrm{Z}_\infty(G, A)$, then $g^\alpha = g$ for every p'-element α of A.

Let G_p be the set of all elements of G such that $g^\alpha = g$ for every p'-element α of A and every $g \in G_p$. Note that if $x, y \in G_p$, then $xy \in G_p$. Hence G_p is a subgroup of G. Let $g \in G_p$, $\alpha, \beta \in A$ and β be a p'-element. Then $\beta^{\alpha^{-1}}$ is a p'-element too. Hence $(g^\alpha)^\beta = g^{\alpha\beta\alpha^{-1}\alpha} = g^{\beta^{\alpha^{-1}}\alpha} = g^\alpha$. It means that $g^\alpha \in G_p$. Thus G_p is an A-admissible subgroup of G. Let x be a p'-element of G_p. From $\mathrm{Inn}G \leq A$ it follows that $\alpha_x : g \to g^x$ is a p'-element of A. Hence it acts trivially on G_p. It means that $x \leq \mathrm{Z}(G_p)$. Let $P \in \mathrm{Syl}_p G_p$. Then $P \trianglelefteq G_p$. So all p-elements of G_p form a subgroup P. Note that P is A-admissible and $N_A(P)/C_A(P) \in \mathfrak{N}_p$. Therefore $P \leq \mathrm{Z}_\infty(G, A)$ by Theorem 1. □

PROOF. [*Proof of Corollary* 5] Let g be a p-element of G. Note that $xg = gx$ is equivalent to $g^x = g$ and $\{\alpha_x : g \to g^x \,|\, x \text{ is a } p'\text{-element of } G\}$ is the set of all p'-elements of $\mathrm{Inn}G$. Now Corollary 5 directly follows from Corollary 4. □

PROOF. [*Proof of Corollary* 6] Directly follows from Proposition 3 and Corollary 4. □

PROOF. [*Proof of Corollary* 7] Assume that G is autonilpotent. Then

$$N_{\mathrm{Aut}G}(P)/C_{\mathrm{Aut}G}(P) \in \mathfrak{N}_p$$

for every p-subgroup P of G and $p \in \pi(G)$ by Corollary 6.

Assume now that $N_{\mathrm{Aut}G}(P)/C_{\mathrm{Aut}G}(P) \in \mathfrak{N}_p$ for every p-subgroup P of G and $p \in \pi(G)$. Suppose that G is non-nilpotent. So there is a Schmidt subgroup S of G. Then S has a normal q-subgroup Q for some prime q and there is a q'-element x of S with $x \notin C_S(Q)$ (see Theorem 26.1 [24, p. 243]). Since $\alpha_x : g \to g^x$ is a non-identity inner automorphism of G of q'-order, $N_{\mathrm{Aut}G}(Q)/C_{\mathrm{Aut}G}(Q) \notin \mathfrak{N}_q$, a contradiction.

Thus G is nilpotent. Hence G is autonilpotent by Proposition 3 and Theorem 1. □

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