ЧЕБЫШЕВСКИЙ СБОРНИК
Том 20. Выпуск 2.

УДК 514.76 DOI 10.22405/2226-8383-2019-20-2-7-21

Обобщенные многообразия Кенмоцу постоянного типа

Абу-Салеем Ахмад, А. Р. Рустанов, Т. Л. Мелехина

Абу-Салеем Ахмад — доцент кафедры математики (геометрия и топология), Университет Аль-Байт (Мафрак, Иордания).
e-mail: abusaleem2@yahoo.com
Рустанов Алигаджи Рабаданович — кандидат физико-математических наук, старший преподаватель кафедры прикладной математики, Национальный исследовательский Московский государственный строительный университет (НИУ МГСУ), Институт фундаментального образования (ИФО МГСУ) (г. Москва).
e-mail: aligadzhi@yandex.ru
Мелехина Татьяна Леонидовна — кандидат физико-математических наук, доцент кафедры анализа данных, принятия решений и финансовых технологий, Финансовый университет при Правительстве Российской Федерации (г. Москва).
e-mail: TMelehina@fa.ru

Аннотация

В работе мы рассматриваем обобщенные многообразия Кенмоцу, мы вводим четвертое и пятое фундаментальные тождества обобщенных многообразий Кенмоцу, вводятся первый и второй структурные тензоры обобщенных многообразий Кенмоцу и доказаны их свойства, вводится понятие присоединенной Q-алгебры для обобщенных многообразий Кенмоцу. Доказано, что обобщенное многообразие Кенмоцу, а также специальные обобщенные многообразия Кенмоцу II рода имеют антикоммутативную присоединенную Q-алгебру. А многообразия Кенмоцу и специальные обобщенные многообразия Кенмоцу I рода имеют абелеву присоединенную Q-алгебру. Вводится контактный аналог постоянства типа и подробно исследуются обобщенные многообразия Кенмоцу постоянного типа. Получены условия точечного постоянства типа обобщенных многообразий Кенмоцу на пространстве присоединенной G-структуры. Доказано, что класс GK-многообразий нулевого постоянного типа совпадает с классом многообразий Кенмоцу, а класс GK-многообразий ненулевого постоянного типа концируклярным преобразованием переводит-ся в почти контактное метрическое многообразие локально эквивалентное произведению шестимерного собственного NK-многообразия на вещественную прямую.

Ключевые слова: многообразия Кенмоцу, обобщенные многообразия Кенмоцу, специальные обобщенные многообразия Кенмоцу I рода, специальные обобщенные многообразия Кенмоцу II рода, GK-многообразия постоянного типа, точнейше косимплектическое многообразие.

Библиография: 15 названий.

Для цитирования:
Абу-Салеем Ахмад, А. Р. Рустанов, Т. Л. Мелехина. Обобщенные многообразия Кенмоцу постоянного типа // Чебышевский сборник, 2019, т. 20, вып. 2, с. 7–21.

CHEBYSHEVSKII SBORNIK
Vol. 20. No. 2.

UDC 514.76 DOI 10.22405/2226-8383-2019-20-2-7-21

Generalized Kenmotsu manifold constancy of type

Abu-Saleem Ahmad, A. R. Rustanov, T. L. Melekhina

Abu-Saleem Ahmad — Professor, Department of Mathematics , Al al-Bayt University (Mafraq, Jordan).
e-mail: abusaleem2@yahoo.com
Rustanov Aligadzhi Rabadanovich — Candidate of Physics and Mathematics, Associate Professor, Department of Applied Mathematics, Federal state budget educational institution of higher education "NATIONAL RESEARCH MOSCOW STATE UNIVERSITY OF CIVIL ENGINEERING"(NRU MGSU) (Moscow).
e-mail: aligadzhi@yandex.ru
Melekhina Tatyana Leonidovna — Candidate of Physics and Mathematics, Associate Professor of Data Analysis, Decision Making and Financial Technologies, Financial University under the Government of the Russian Federation (Moscow).
e-mail: TMelehina@fa.ru

Abstract

In this work we consider generalized Kenmotsu manifolds, we introduce: the fourth and the fifth fundamental identities of generalized Kenmotsu manifolds; the first and the second structural tensors of generalized Kenmotsu manifolds (and we prove their properties); the concept of adjoint Q-algebra for generalized Kenmotsu manifolds. We prove that generalized Kenmotsu manifolds and the II kind special generalized Kenmotsu manifolds have anticommutative adjoint Q-algebra. And the Kenmotsu manifolds and the I kind special generalized Kenmotsu manifolds have Abelian adjoint Q-algebra. The type constancy contact analog is introduced and the constant-type generalized Kenmotsu manifolds are thoroughly examined. We have identified the type point constancy conditions of the generalized Kenmotsu manifolds in the adjoint G-structure space. We prove that the zero constant type GK-manifold class coincides with the Kenmotsu manifold class and the non-zero constant type GK-manifold class can be concircularly transformed into the almost contact metric manifolds locally equivalent to the product of the six dimensional NK-eigenmanifold and the real straight line.

Keywords: Kenmotsu manifolds, generalized Kenmotsu manifolds, the I kind special generalized Kenmotsu manifolds, the II kind special generalized Kenmotsu manifolds, constant type GK-manifolds, most precise cosymplectic manifold.

Bibliography: 15 titles.

For citation:
Abu-Saleem Ahmad, A. R. Rustanov, T. L. Melekhina, 2019, "Generalized Kenmotsu manifold constancy of type" , *Chebyshevskii sbornik*, vol. 20, no. 2, pp. 7–21.

1. Introduction

Contact and almost contact structures are one of the most substantial examples of differential geometrical structures. Nevertheless the most important geometrical properties of almost contact metric manifolds are revealed when the additional limitations are applied to them. The most natural limitation is the isotropy condition.

The almost Hermitian manifold isotropy can be characterized by the constancy of their type [1], [2]. The type constancy of approximately Keller manifolds was introduced by A. Gray [2] and proved to be very useful for approximately Keller manifold studying. The complete characteristics of approximately Keller manifolds were obtained by V.F. Kirichenko [3].

In this work we consider the type constancy contact analog for generalized Kenmotsu manifolds which were introduced in the thesis work of Umnova S.V. [4]. In the work [4] Umnova S.V. singles out two subsets of generalized Kenmotsu manifolds, called special generalized Kenmotsu manifolds (shorter, SGK-) of the I and II kind. In the work [4] it's proved that generalized Kenmotsu manifolds of the constant curvature are the Kenmotsu manifolds [5] of the constant curvature (-1). Moreover, it's proved that the class of SGK- manifolds of the II kind coincides with the class of almost contact metrical manifolds received from the most precise cosymplectic manifolds [6] through canonical transformation of the most precise cosymplectic structure and the local construction of these manifolds is given. In this article we explore the generalized Kenmotsu manifolds of the constant type and give their complete local characteristics.

This article is organized in the next way. In Section 2 we present the preliminaries for the next statements, build the space of adjoint G-structure and put down the first structural equation group on the adjoint G-structure space. In Section 3 we give the definition of generalized Kenmotsu manifolds, give the full structural equation group, we prove that a generalized Kenmotsu manifold in a dimension different from 5 is a special generalized Kenmotsu manifold of the II kind and we provide fundamental identities of generalized Kenmotsu manifolds.

In the Section 4 we consider the adjoint Q-algebra of a generalized Kenmotsu manifold. We establish the theorem which is the basic result of the present paragraph and which means that the adjoint Q-algebra of a generalized Kenmotsu manifold is anticommutative. Three Conclusions are given for this theorem; they characterize the adjoint Q-algebras of generalized Kenmotsu manifold special cases.

In the Section 5 we explore the generalized Kenmotsu manifolds of a constant type. It's proved that the generalized Kenmotsu manifolds of the non-zero constant type are the generalized Kenmotsu manifolds of the II kind and we received their local structure. Generalized Kenmotsu manifolds of the non-zero type coincide with the Kenmotsu manifolds.

2. Preliminaries

Assume, that M is a smooth manifold of dimension $2n+1$, $\mathcal{X}(M)$ – C^∞ is a module of smooth vector fields on the manifold M. Further all manifolds, tensor fields and the like are supposed to be smooth of the class C^∞.

Definition 1. *[7] An almost contact structure on the manifold M is the triplet (η, ξ, Φ) of tensor fields on this manifold where η is a differential 1-form which is called a contact form of structure, ξ is a vector field which is called characteristical, Φ is the endomorphism of module $\mathcal{X}(M)$ which is called structural endomorphism. Here*

$$1)\ \eta(\xi) = 1;\ 2)\ \eta \circ \Phi = 0;\ 3)\ \Phi(\xi) = 0;\ 4)\ \Phi^1 = -id + \eta \otimes \xi. \tag{1}$$

Besides, if such a Riemannian structure $g = \langle \cdot, \cdot \rangle$ is fixed on M that

$$\langle \Phi X, \Phi Y \rangle = \langle X, Y \rangle - \eta(X)\eta(Y), \quad X, Y \in \mathcal{X}(M). \tag{2}$$

quadruple $(\eta, \xi, \Phi\, g = \langle \cdot, \cdot \rangle)$ is called *almost contact metrical (shorter, AC-) structure*. The manifold where an almost contact (metrical) structure is fixed is called an *almost contact (metrical (shorter, AC-)) manifold.*

Skew-symmetric tensor $\Omega(X,Y) = \langle X, \Phi Y\rangle, X, Y \in \mathcal{X}(M)$ is called *the fundamental form of an AC-structure* [7].

Assume, that $(\eta, \xi, \Phi\, g = \langle\cdot,\cdot\rangle)$ is an almost contact metrical structure on the manifold M^{2n+1}. In the module $\mathcal{X}$(M) two mutually complementing projections $m = \eta \otimes \xi$ and $l = id - m = -\Phi^2$ [8]; are internally defined in the way that $\mathcal{X}$(M)=$\mathcal{L} \oplus \mathcal{M}$, where $\mathcal{L} = Im(\Phi) = ker\eta$ – a so-called contact distribution, dim$\mathcal{L}$=2n, $\mathcal{M} = Imm = ker(\Phi) = L(\xi)$ – a linear span of a structure vector (also l and m are the projectors for submodules $\mathcal{L}$, $\mathcal{M}$ accordingly). Obviously that distributions $\mathcal{L}$ and $\mathcal{M}$ are invariant towards Φ and are mutually orthogonal. It's also obvious that $\tilde{\Phi}^2 = -id$, $\left\langle \tilde{\Phi}X, \tilde{\Phi}Y \right\rangle = \langle X, Y\rangle$, X,Y$\in$ $\mathcal{X}$(M), where $\tilde{\Phi} = \Phi|_{\mathcal{L}}$. Consequently $\{\tilde{\Phi}_p, g_p|_{\mathcal{L}}\}$ is a Hermitian structure on the space $\mathcal{L}_p$ (p – some point of M).

The complexification $\mathcal{X}(\mathrm{M})^C$ of the module $\mathcal{X}$(M) disintegrates into a direct sum

$$\mathcal{X}(M)^C = D_{\Phi}^{\sqrt{-1}} \oplus D_{\Phi}^{-\sqrt{-1}} \oplus D_{\Phi}^{0}$$

of structural endomorphism Φ own spaces corresponding to their own values $\sqrt{-1}$, $-\sqrt{-1}$ and 0 accordingly. Besides, the projectors for the summands of this direct sum will be endomorphisms ([7], [8]): $\pi = \sigma \circ l = -\frac{1}{2}(\Phi^2 + \sqrt{-1}\Phi)$, $\bar{\pi} = \bar{\sigma} \circ l = \frac{1}{2}(-\Phi^2 + \sqrt{-1}\Phi)$, $m = id + \Phi^2$, where $\sigma = \frac{1}{2}(id - \sqrt{-1}\Phi)$, $\bar{\sigma} = \frac{1}{2}(id + \sqrt{-1}\Phi)$.

The depictions $\sigma_p : \mathcal{L}_p \to D_{\Phi}^{\sqrt{-1}}$ and $\bar{\sigma}_p : \mathcal{L}_p \to D_{\Phi}^{-\sqrt{-1}}$ are the isomorphism and the anti-isomorphism of Hermitian spaces, accordingly. That's why to every point $p \in M^{2n+1}$ it's possible to add a family of reference frames of space $T_p(M)^C$ of the kind $(p, \epsilon_0, \epsilon_1, \ldots, \epsilon_n, \epsilon_{\hat{1}}, \ldots, \epsilon_{\hat{n}})$ where $\epsilon_a = \sqrt{2}\sigma_p(e_a)$, $\epsilon_{\hat{a}} = \sqrt{2}\bar{\sigma}_p(e_a)$, $\sigma_0 = \xi_p$ where $\{e_a\}$ is the orthonormalized base of the Hermitian space $\mathcal{L}_p$. This reference frame is called an A-reference frame [8]. It's clear that matrices are the element of tensors Φ_p and g_p in the A-reference frame; they have the according forms of:

$$(\Phi^i_j) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sqrt{-1}I_n & 0 \\ 0 & 0 & -\sqrt{-1}I_n \end{pmatrix}, (g_{ij}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & I_n \\ 0 & I_n & 0 \end{pmatrix}, \tag{3}$$

where I_n is the unity matrix of the n-order. It is well known [7], [8] that the aggregate of such reference frames determines the G-structure on M with structural group $\{1\} \times U(n)$ represented by such matrices as $\begin{pmatrix} 1 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \end{pmatrix}$, where $A \in U(n)$. This G-structure is called *adjoint* [7], [8].

Assume that $(M^{2n+1}, \eta, \xi, \Phi\, g = \langle\cdot,\cdot\rangle)$ is an almost contact metrical manifold. We will make a convention that in this entire work indices i, j, k, l run through values from 1 to $2n$, indices a, b, c, d, – through values from 1 to n, and let's assume that $\hat{a} = a + n$, $\hat{\hat{a}} = a$, $\hat{0} = 0$, unless it is stated otherwise. Let (U, φ) be a local chart on manifold M. According to the tensor analysis fundamental theorem the assignment of the structural endomorphism Φ and the Riemann structure $g = \langle\cdot,\cdot\rangle$ on the manifold M inspires on the total BM space of a bundle of frames above M a system of functions $\{\Phi^i_j\}$, $\{g_{ij}\}$, complying in the coordinate neighborhood $W = \pi^{-1}(U) \subset BM$ with a differential equation system of the following form

$$d\Phi^i_j + \Phi^k_j\theta^i_k - \Phi^i_k\theta^k_j = \Phi_(j,k)^i\omega^k, \quad dg_{ij} - g_kj\theta^k_i - g_{ik}\theta^k_j = g_{ij,k}\omega^k, \tag{4}$$

where $\{\omega^i\}$, $\{\theta^i_j\}$ are the components of the solder forms and the Riemannian connection, correspondingly; $\Phi^i_{j,k}$, $g_{ij,k}$ are the components of the covariant tensor differential Φ and g in this connection correspondingly. Moreover, because of the Riemannian connection definition $\nabla g = 0$, and it means that

$$g_{ij,k} = 0. \tag{5}$$

With account of (3) and (5) relations (4) on the adjoint G-structure space can be put down as [7], [8]

$$\begin{gathered}
\Phi^a_{b,k}=0,\quad \Phi^{\hat a}_{\hat b,k}=0,\quad \Phi^0_{0,k}=0,\\
\theta^a_{\hat b}=\tfrac{\sqrt{-1}}{2}\Phi^a_{\hat b,k}\omega^k,\quad \theta^{\hat a}_b=-\tfrac{\sqrt{-1}}{2}\Phi^{\hat a}_{b,k}\omega^k,\\
\theta^a_0=\sqrt{-1}\Phi^a_{0,k}\omega^k,\quad \theta^{\hat a}_0=-\sqrt{-1}\Phi^{\hat a}_{0,k}\omega^k,\\
\theta^0_a=-\sqrt{-1}\Phi^0_{a,k}\omega^k,\quad \theta^0_{\hat a}=\sqrt{-1}\Phi^0_{\hat a,k}\omega^k,\\
\theta^i_j+\theta^{\hat i}_{\hat j}=0,\quad \theta^0_0=0.
\end{gathered}\tag{6}$$

Besides, note that because of the real type nature of the corresponding forms and tensors $\overline{\omega^i}=\omega^{\hat i}$, $\overline{\theta^i_j}=\theta^{\hat i}_{\hat j}$, $\overline{\nabla\Phi^i_{j,k}}=\nabla\Phi^{\hat i}_{\hat j,\hat k}$, where $t\to\bar t$ is a complex conjugation operator.

With regard to the obtained relations the first structural equation group of the Riemannian connection $d\omega^i=-\theta^i_j\wedge\omega^j$ on the adjoint G-structure space of the almost contact metrical manifold can be formulated as the first group of almost contact metrical manifold structural equations [7], [8]:

$$\begin{gathered}
d\omega=C_{ab}\omega^a\wedge\omega^b+C^{ab}\omega_a\wedge\omega_b+C^b_a\omega^a\wedge\omega_b+C_a\omega\wedge\omega^a+C^a\omega\wedge\omega_a;\\
d\omega^a=-\theta^a_b\wedge\omega^b+B^{ab}{}_c\omega^c\wedge\omega_b+B^{abc}\omega_b\wedge\omega_c+B^{ab}{}_b+B^a{}_b\omega\wedge\omega^b;\\
d\omega_a=\theta^b_a\wedge\omega_b+B_{ab}{}^c\omega_c\wedge\omega^b+B_{abc}\omega^b\wedge\omega^c+B_{ab}\omega\wedge\omega^b+B_a{}^b\omega\wedge\omega_b,
\end{gathered}\tag{7}$$

where $\omega=\omega^0=\pi^*(\eta)$; π is a natural projection of the adjoint G-structure space on the manifold M, $\omega_i=g_{ij}\omega^i$,

$$\begin{gathered}
B^{ab}{}_c=-\tfrac{\sqrt{-1}}{2}\Phi^a_{\hat b,c},\quad B_{ab}{}^c=\tfrac{\sqrt{-1}}{2}\Phi^{\hat a}_{b,\hat c},\quad B^{abc}=\tfrac{\sqrt{-1}}{2}\Phi^a_{\hat b,\hat c},\\
B_{abc}=-\tfrac{\sqrt{-1}}{2}\Phi^{\hat a}_{b,c},\quad B^a{}_b=\sqrt{-1}\Phi^a_{0,b},\quad B_a{}^b=-\sqrt{-1}\Phi^{\hat a}_{0,\hat b},\\
B^{ab}=\sqrt{-1}(\Phi^a_{0,\hat b}-\tfrac12\Phi^a_{\hat b,0}),\quad B_{ab}=-\sqrt{-1}(\Phi^{\hat a}_{0,b}-\tfrac12\Phi^{\hat a}_{b,0}),\\
C^{ab}=\sqrt{-1}\Phi^0_{[\hat a,\hat b]},\quad C_{ab}=-\sqrt{-1}\Phi^0_{[a,b]},\quad C^b_a=-\sqrt{-1}(\Phi^0_{\hat b,a}+\Phi^0_{a,\hat b}),\\
C^a=-\sqrt{-1}\Phi^0_{\hat a,0},\quad C_a=\sqrt{-1}\Phi^0_{a,0}.
\end{gathered}\tag{8}$$

3. Generalized Kenmotsu manifolds

Assume that $(M^{2n+1},\Phi,\xi,\eta,g=\langle\cdot,\cdot\rangle)$ is an almost contact metrical manifold.

DEFINITION 2. *[4]. The class of almost contact metrical manifolds characterized by equality*

$$\nabla_X(\Phi)Y+\nabla_Y(\Phi)X=-\eta(Y)\Phi X-\eta(X)\Phi Y;\quad X,Y\in\mathcal{X}(M),\tag{9}$$

is called generalized Kenmotsu manifolds (shorter, GK- manifolds).

The full group of GK-manifold structural equations takes the form [9]:

$$\begin{gathered}
1)\ d\omega=F_{ab}\omega^a\wedge\omega^b+F^{ab}\omega_a\wedge\omega_b;\\
2)\ d\omega^a=-\theta^a_b\wedge\omega^b+C^{abc}\omega_b\wedge\omega_c-\tfrac32F^{ab}\omega\wedge\omega_b+\delta^a_b\omega\wedge\omega^b;\\
3)\ d\omega_a=\theta^b_a\wedge\omega_b+C_{abc}\omega^b\wedge\omega^c-\tfrac32F_{ab}\omega\wedge\omega^b+\delta^b_a\omega\wedge\omega_b;\\
4)\ d\theta^a_b=-\theta^a_c\wedge\theta^c_b+(A^{ad}_{bc}-2C^{adh}C_{hbc}-\tfrac32F^{ad}F_{bc})\omega^c\wedge\omega_d+\\
+(-\tfrac13\delta^a_bF_{cd}+\tfrac23\delta^a_cF_{db}+\tfrac23\delta^a_dF_{bc})\omega^c\wedge\omega^d+(\tfrac13\delta^a_bF^{cd}-\tfrac23\delta^c_bF^{da}-\tfrac23\delta^d_bF^{ac})\omega_c\wedge\omega_d;
\end{gathered}$$

$$\begin{gathered}
5)\ dC^{abc}+C^{dbc}\theta_d^a+C^{adc}\theta_d^b+C^{abd}\theta_d^c=C^{abcd}\omega_d-2\delta_d^{[a}F^{bc]}\omega^d-C^{abc}\omega;\\
6)\ dC_{abc}-C_{dbc}\theta_a^d-C_{adc}\theta_b^d-C_{abd}\theta_c^d=C_{abcd}\omega^d-2\delta_{[a}^dF_{bc]}\omega_d-C_{abc}\omega;\\
7)\ dF^{ab}+F^{cb}\theta_c^a+F^{ac}\theta_c^b=-2F^{ab}\omega;\\
8)\ dF_{ab}-F_{cb}\theta_a^c-F_{ac}\theta_b^c=-2F_{ab}\omega;
\end{gathered} \tag{10}$$

where

$$\begin{gathered}
C^{abc}=\frac{\sqrt{-1}}{2}\Phi^a_{\hat{b},\hat{c}};\quad C_{abc}=-\frac{\sqrt{-1}}{2}\Phi^{\hat{a}}_{b,c};\quad C^{[abc]}=C^{abc};\quad C_{[abc]}=C_{abc};\\
\overline{C^{abc}}=C_{abc};\quad F^{ab}=\sqrt{-1}\Phi^0_{\hat{a},\hat{b}};\quad F_{ab}=-\sqrt{-1}\Phi^0_{a,b};\\
F^{ab}+F^{ba}=0;\quad F_{ab}+F_{ba}=0;\quad \overline{F^{ab}}=F_{ab};\quad A^{ad}_{[bc]}=A^{[ad]}_{bc}=0;\\
C^{a[bcd]}=\tfrac{3}{2}F^{a[b}F^{cd]};\quad C_{a[bcd]}=\tfrac{3}{2}F_{a[b}F_{cd]};\quad F_{ad}C^{dbc}=F^{ad}C_{dbc}=0.
\end{gathered}$$

PROPOSITION 1. *[9]. If $C^{abc}=C_{abc}=0$ and $F^{ab}=F_{ab}=0$, then a GK-manifold is a Kenmotsu manifold.*

Differentiating externally (10_4 - 10_6) we get:

$$\begin{gathered}
1)\ dA^{ad}_{bc}+A^{hd}_{bc}\theta^a_h+A^{ah}_{bc}\theta^d_h-A^{ad}_{hc}\theta^h_b-A^{ad}_{bh}\theta^h_c=A^{ad}_{bch}\omega^h+A^{adh}_{bc}\omega_h+A^{ad}_{bc0}\omega;\\
2)\ dC^{abcd}+C^{hbcd}\theta^a_h+C^{ahcd}\theta^b_h+C^{abhd}\theta^c_h+C^{abch}\theta^d_h=\\
=C^{abcdh}\omega_h+C^{abcd0}\omega;\\
3)\ dC_{abcd}-C_{hbcd}\theta^h_a-C_{ahcd}\theta^h_b-C_{abhd}\theta^h_c-C_{abch}\theta^h_d=\\
=C_{abcdh}\omega^h+C_{abcd0}\omega.
\end{gathered} \tag{11}$$

Herewith:

$$\begin{gathered}
1)\ A^{ad}_{b[ch]}=0;\quad 2)\ A^{a[dh]}_{bc}=0;\\
3)\ A^{ad}_{bc0}=-2A^{ad}_{bc}+F^{ad}F_{bc}-2\delta^a_bF^{dh}F_{hc}-2\delta^a_cF^{dh}F_{hb}-2\delta^d_bF^{ah}F_{hc};\\
4)\ (A^{ag}_{b[c}-2C^{agf}C_{fb[c})C_{|g|dh]}=0;\quad 5)\ (A^{a[c}_{bg}-2C^{a[c|f|}C_{fbg})C^{|g|dh]}=0;\\
6)\ (A^{ah}_{b[c}-\tfrac{3}{2}F^{ah}F_{b[c})F_{|h|d]}=0;\quad 7)\ (A^{a[d}_{bh}-\tfrac{3}{2}F^{a[d}F_{bh})F^{|h|c]}=0;\\
8)\ C^{abc[dh]}=-2\{C^{abc}F^{dh}+\tfrac{1}{3}(C^{adh}F^{bc}+C^{bdh}F^{ca}+C^{cdh}F^{ab}+\\
+C^{abh}F^{dc}+C^{acd}F^{hb}+C^{ahc}F^{db}+C^{dbc}F^{ah}+C^{hbc}F^{da})\};\\
9)\ C^{abcd0}=-(2C^{abcd}+F^{ab}F^{cd}+F^{ac}F^{db}+F^{ad}F^{bc});\\
10)\ C^{abcg}C_{gdh}=0;\quad 11)\ C^{abch}F_{hd}=0;\\
12)\ C_{abc[dh]}=-2\{C_{abc}F_{dh}+\tfrac{1}{3}(C_{adh}F_{bc}+C_{bdh}F_{ca}+C_{cdh}F_{ab}+C_{dbc}F_{ah}+\\
+C_{hbc}F_{da}+C_{acd}F_{hb}+C_{ahc}F_{db}+C_{abd}F_{ch}+C_{abh}F_{dc})\};\\
13)\ C_{abcd}{}^h=(A^{gh}_{ad}-2C^{ghf}C_{fad})C_{gbc}+(A^{gh}_{bd}-2C^{ghf}C_{fbd})C_{agc}+\\
+(A^{gh}_{cd}-2C^{ghf}C_{fcd})C_{abg};\\
14)\ C_{abcd0}=-(2C_{abcd}+F_{ab}F_{cd}+F_{ac}F_{db}+F_{ad}F_{bc});\\
15)\ C_{abcg}C^{gdh}=0;\quad 16)\ C_{abch}F^{hd}=0.
\end{gathered} \tag{12}$$

Differentiating externally (10_7) and (10_8), we get:

$$\begin{gathered}1)\ A^{ad}_{hc}F^{hb}-A^{bd}_{hc}F^{ha}=\tfrac{3}{2}F^{ad}F_{hc}F^{hb}-\tfrac{3}{2}F^{bd}F_{hc}F^{ha};\\ 2)\ 2F^{ab}F_{cd}=(\delta^a_dF_{ch}-\delta^a_cF_{dh})F^{hb}+(\delta^b_cF_{dh}-\delta^b_dF_{ch})F^{ha};\\ 3)\ 2F^{ab}F^{cd}=F^{ac}F^{db}+F^{ad}F^{bc};\\ 4)\ F_{ah}A^{hd}_{bc}-F_{bh}A^{hd}_{ac}=\tfrac{3}{2}(F_{ad}F_{bh}-F_{bd}F_{ah})F^{hc};\\ 5)\ 2F_{ab}F^{cd}=(\delta^c_bF^{dh}-\delta^d_bF^{ch})F_{ha}+(\delta^d_aF^{ch}-\delta^c_aF^{dh})F_{hb};\\ 6)\ 2F_{ab}F_{cd}=F_{ac}F_{db}+F_{ad}F_{bc}.\end{gathered}\tag{13}$$

As a useful consequence (13_2) we prove the next theorem.

THEOREM 1. *[9] A GK-manifold of dimension different from 5 is a SGK-manifold of the II kind.*

The identity

$$(A^{ag}_{b[c}-2C^{agf}C_{fb[c})C_{|g|dh]}=0\tag{14}$$

we call **the first fundamental equality of GK-manifolds** [10].

The identity

$$(A^{ah}_{b[c}-\tfrac{3}{2}F^{ah}F_{b[c})F_{|h|d]}=0\tag{15}$$

we call **the second fundamental equality of GK-manifolds** [10].

The identity

$$2F^{ab}F^{cd}=F^{ac}F^{db}+F^{ad}F^{bc}\tag{16}$$

we call **the third fundamental equality of GK-manifolds** [10].

The identity

$$F_{ad}C^{dbc}=F^{ad}C_{dbc}=0\tag{17}$$

we call **the forth fundamental equality of GK-manifolds**.

The identity

$$C_{abcg}C^{gdh}=0\tag{18}$$

we call **the fifth fundamental equality of GK-manifolds**.

The system of functions (C^{abc},C_{abc}) determines the tensor (2,1) which is called **the first structural tensor**, the system of functions (F^{ab},F_{ab}) determines the tensor (1,0) which is called **the second structural tensor**. The structural tensors of a GK-structure have the following equations [10]:

$$\begin{gathered}1)\ C(X,Y)=-\tfrac{1}{2}\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X=-\tfrac{1}{2}\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi^2X;\\ 2)\ F(X)=\Phi\circ\nabla_{\Phi^2X}(\Phi)\xi-\Phi^2X=-\Phi\circ\nabla_X(\Phi)\xi-\Phi^2X=-\nabla_X\xi-\Phi^2X=\\ =-\Phi^2\circ\nabla_{\Phi X}(\Phi)\xi-\Phi^2X=-\Phi\circ\nabla_{\Phi X}(\Phi)\xi-\Phi^2X;\quad X,Y\in\mathcal{X}(M).\end{gathered}\tag{19}$$

DEFINITION 3. *[4] A generalized Kenmotsu manifold with a zero first structural tensor is called **a special generalized Kenmotsu manifold** (SGK-, for short) **of the I kind**.*

DEFINITION 4. *[4] A generalized Kenmotsu manifold with a zero second structural tensor is called **a special generalized Kenmotsu manifold** (SGK-, for short) **of the II kind**.*

4. Q-algebras of generalized Kenmotsu manifolds

In this section we discuss the Q-algebra adjoint to a GK-manifold.

Definition 5. *[11] **A Q-algebra** is a triplet $\{V, \langle\langle\cdot,\cdot\rangle\rangle, *\}$ where V is a module of the commutative associative ring **K** with nontrivial involution; $\langle\langle\cdot,\cdot\rangle\rangle$ is a non-degenerated Hermitian form on V; * is a binary operation $*: V \times V \to V$, antilinear for each argument for which the Q-algebra axiom is accomplished* $\langle\langle X * Y, Z\rangle\rangle + \overline{\langle\langle Z * X, Y\rangle\rangle} = 0,\ X, Y, Z \in V$.

If **K**=**C**, then Q-algebra V is called **complex**.

Definition 6. *[12] Q-algebra V is called:*
- ***Abelian**, or **commutative Q-algebra**, if* $X * Y = 0$, $(X, Y \in V)$;
- ***K-algebra**, or **anti-commutative Q-algebra**, if* $X * Y = -Y * X$, $(X, Y \in V)$;
- ***A-algebra**, or **pseudo-commutative Q-algebra**, if*

$$\langle X * Y, Z\rangle + \langle Y * Z, X\rangle + \langle Z * X, Y\rangle = 0, \quad (X, Y, Z \in V).$$

We recall [13] that in the module $\mathcal{X}$(M) of an almost contact metrical manifold the structure of Q-algebra Re is naturally introduced over the ring of complex-valued smooth functions with the operation

$$X * Y = T(X, Y) = \frac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\}; \quad X, Y \in \mathcal{X}(M) \tag{20}$$

and metrics

$$\langle\langle X, Y\rangle\rangle = \langle X, Y\rangle + \sqrt{-1}\,\langle X, \Phi Y\rangle\,; X, Y \in \mathcal{X}(M). \tag{21}$$

This Q-algebra is called adjoint.

Assume that M is a GK-manifold. In the $C^\infty(M)$ — module $\mathcal{X}$(M) of smooth vector fields of manifold M a binary operation "$*$" is introduced by the formula

$$X * Y = T(X, Y) = \frac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\}; \quad X, Y \in \mathcal{X}(M)$$

Theorem 2. *The GK-structure has an anti-commutative adjoint Q-algebra, h.e. a K-algebra.*

Proof. From Definition 2 it is easy to follow that $\Phi\nabla_{\Phi X}(\Phi)\Phi Y + \Phi\nabla_{\Phi Y}(\Phi)\Phi X$; $X, Y \in \mathcal{X}(M)$, so $\Phi\nabla_{\Phi X}(\Phi)\Phi Y = -\Phi\nabla_{\Phi Y}(\Phi)\Phi X$; $X, Y \in \mathcal{X}(M)$.

And it means that $\Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y = -\Phi\nabla_{\Phi^2 Y}(\Phi)\Phi^2 X$; $X, Y \in \mathcal{X}(M)$. Then

$$T(X, Y) = \tfrac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\} =$$
$$= -\tfrac{1}{4}\{\Phi\nabla_{\Phi Y}(\Phi)\Phi X - \Phi\nabla_{\Phi^2 Y}(\Phi)\Phi^2 X\} = -T(Y, X); \ X, Y \in \mathcal{X}(M).$$

H.e. the adjoint Q-algebra of GK-structures is a K-algebra.

Corollary 1. *SGK-manifolds of the I kind and Kenmotsu manifolds have **Abelian** adjoint Q-algebra.*

Proof. For SGK-manifolds of the I kind and Kenmotsu manifolds the identity is realized ([9]) $\Phi\nabla_{\Phi X}(\Phi)\Phi Y = -\Phi\nabla_{\Phi Y}(\Phi)\Phi X = 0$; $X, Y \in \mathcal{X}(M)$, and it means that

$$T(X, Y) = \frac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\} = 0;$$

$X, Y \in \mathcal{X}(M)$, h.e. adjoint Q-algebra is Abelian.

Corollary 2. *SGK-manifolds of the II kind have anti-commutative adjoint Q-algebra.*

Proof. From (9) we have $\Phi\nabla_{\Phi X}(\Phi)\Phi Y + \Phi\nabla_{\Phi Y}(\Phi)\Phi X$; $X, Y \in \mathcal{X}(M)$, than according to the obtained equality

$$\begin{aligned} X * Y = T(X,Y) = \tfrac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\} = \\ = -\tfrac{1}{4}\{\Phi\nabla_{\Phi Y}(\Phi)\Phi X - \Phi\nabla_{\Phi^2 Y}(\Phi)\Phi^2 X\} = \\ = -T(Y,X) = -X * Y;\ X, Y \in \mathcal{X}(M), \end{aligned}$$

h.e. the adjoint Q-algebra is commutative, h.e. a K-algebra.

Corollary 3. *Kenmotsu manifolds have an Abelian adjoint Q-algebra.*

Proof. For Kenmotsu manifolds the equality is executed ([11])

$$T(X,Y) = \frac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\} = 0, X, Y \in \mathcal{X}(M),$$

h.e. the adjoint Q-algebra is Abelian.

5. The type constancy of generalized Kenmotsu manifolds

In this section we consider a contact analog of the type constancy and examine it in detail for generalized Kenmotsu manifolds.

Definition 7. *[14] The complex K-algebra* Re *is called the K-algebra of constant type, if* $\exists c \in C \ \ \forall X, Y \in \mathrm{Re} : \langle\langle X, Y\rangle\rangle = 0 \Rightarrow \|X * Y\|^2 = c\,\|X\|^2\,\|Y\|^2$.

Definition 8. *The GK-manifold M is called* ***a pointlike constant type*** *manifold, if its adjoint Q-algebra has a constant type in each point of manifold M. Function c, if it exists, is called* ***the type constant*** *of the GK-manifold. If c = const, than M is called* ***a global constant type*** *GK-manifold.*

Assume that M is a GK-manifold. Let us consider Q-algebra Re, adjoint to manifold M, with operation $* : \mathcal{X}(M) \times \mathcal{X}(M) \to \mathcal{X}(M)$, defined by the identity

$$X * Y = T(X,Y) = \frac{1}{4}\{\Phi\nabla_{\Phi X}(\Phi)\Phi Y - \Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\};$$

$X, Y \in \mathcal{X}(M)$. From (9) it follows that on the GK-manifold $\Phi\nabla_{\Phi X}(\Phi)\Phi Y = -\Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y$; $X, Y \in \mathcal{X}(M)$. Thus $X * Y = \frac{1}{2}\Phi\nabla_{\Phi X}(\Phi)\Phi Y = -\frac{1}{2}\Phi\nabla_{\Phi^2 X}(\Phi)\Phi^2 Y\}$; $X, Y \in \mathcal{X}(M)$. Because of (21), the condition $\langle X, Y\rangle = \langle X, \Phi Y\rangle = 0$ equals the condition $\langle\langle X, Y\rangle\rangle = 0$. Thus, the following is true

Theorem 3. *A GK-manifold is a manifold of pointlike constant type c then and only then, when*

$$\forall X, Y \in \mathcal{X}(M)\ \ \langle\langle X, Y\rangle\rangle = 0 \Rightarrow \|C(X,Y)\|^2 = c\,\|X\|^2\,\|Y\|^2. \tag{22}$$

We introduce into consideration a 4-form

$$\mathcal{C}(X,Y,Z,W) = \langle\langle X * Y, Z * W\rangle\rangle = \langle\langle C(X,Y), C(Z,W)\rangle\rangle.$$

It is directly verified that it has the following properties:

1) Antilinearity at the first pair of arguments

$$\sqrt{-1}\mathcal{C}(X,Y,Z,W) = -\mathcal{C}(\Phi X, Y, Z, W) = -\mathcal{C}(X, \Phi Y, Z, W).$$

2) Linearity at the second pair of arguments

$$\sqrt{-1}\mathcal{C}(X,Y,Z,W) = -\mathcal{C}(X,Y,\Phi Z,W) = -\mathcal{C}(X,Y,Z,\Phi W).$$

3) Skew symmetry at the first and second pairs of arguments

$$\mathcal{C}(X,Y,Z,W) = -\mathcal{C}(Y,X,Z,W) = -\mathcal{C}(X,Y,W,Z).$$

4) Hermicity

$$\mathcal{C}(X,Y,Z,W) = \overline{\mathcal{C}(Z,W,X,Y)}, X,Y,Z,W \in \mathcal{X}(M).$$

Because

$$\mathcal{C}(X,Y,X,Y) = \langle\langle X*Y, X*Y\rangle\rangle = \langle\langle C(X,Y), C(X,Y)\rangle\rangle = \|C(X,Y)\|^2,$$

GK-manifold M is of a pointlike constant type c then and only then, when

$$\mathcal{C}(X,Y,X,Y) = c\,\|X\|^2\,\|Y\|^2,\ \ X,Y \in \mathcal{X}(M),\quad \langle\langle X,Y\rangle\rangle = 0. \tag{23}$$

We polarize this equality, replacing Y with $Y+Z$, where $Z \in \mathcal{X}(M)$, $\langle\langle X,Z\rangle\rangle = 0$: $\mathcal{C}(X,Y+Z,X,Y+Z) = c\,\|X\|^2\,\|Y+Z\|^2$. After distribution through its linearity and the required reduction considering (23), we get

$$\mathcal{C}(X,Y,X,Z) + \mathcal{C}(X,Z,X,Y) = c\,\|X\|^2\,(\langle\langle Y,Z\rangle\rangle + \langle\langle Z,Y\rangle\rangle). \tag{24}$$

Replacing Z with ΦZ here, while considering the properties 1) and the non-degeneracy of endomorphism Φ we get:

$$\mathcal{C}(X,Y,X,Z) - \mathcal{C}(X,Z,X,Y) = c\,\|X\|^2\,(-\langle\langle Y,Z\rangle\rangle + \langle\langle Z,Y\rangle\rangle). \tag{25}$$

Summing the identities (24) and (25) term by term we get:

$$\mathcal{C}(X,Y,X,Z) = c\,\|X\|^2\,\langle\langle Z,Y\rangle\rangle. \tag{26}$$

Let now $Y,Z \in \mathcal{X}(M)$ be arbitrary vectors. Let us distribute them over the linear hull of vector X and its orthogonal complement: $Y = \frac{\langle\langle Y,X\rangle\rangle}{\|X\|^2}X + Y'$; $Z = \frac{\langle\langle Z,X\rangle\rangle}{\|X\|^2}X + Z'$. Considering (26) and the property 3) after the required reduction we get:

$$\begin{aligned}\mathcal{C}(X,Y,X,Z) = \mathcal{C}(X,Y',X,Z') = c\,\|X\|^2\,\langle\langle Z',Y'\rangle\rangle = \\ = c\,\|X\|^2\left\langle\left\langle Z - \tfrac{\langle\langle Z,X\rangle\rangle}{\|X\|^2}X, Y - \tfrac{\langle\langle Y,X\rangle\rangle}{\|X\|^2}X\right\rangle\right\rangle = \\ = c\{\langle\langle Z,Y\rangle\rangle\,\|X\|^2 - \langle\langle Z,X\rangle\rangle\,\langle\langle X,Y\rangle\rangle\}.\end{aligned}$$

So,

$$\mathcal{C}(X,Y,X,Z) = c\{\langle\langle Z,Y\rangle\rangle\,\|X\|^2 - \langle\langle Z,X\rangle\rangle\,\langle\langle X,Y\rangle\rangle\}. \tag{27}$$

Let us replace Z with W in the obtained equality, then

$$\mathcal{C}(X,Y,X,W) = c\{\langle\langle W,Y\rangle\rangle\,\|X\|^2 - \langle\langle W,X\rangle\rangle\,\langle\langle X,Y\rangle\rangle\}.$$

In the last identity we replace X with $X+Z$ and after removal through linearity and after the required reduction while considering (27) we get:

$$\mathcal{C}(X,Y,Z,W) = c\{\langle\langle W,Y\rangle\rangle\,\langle\langle Z,X\rangle\rangle - \langle\langle W,X\rangle\rangle\,\langle\langle Z,Y\rangle\rangle\}. \tag{28}$$

Inverse, it is obvious that because (28), (23) is fulfilled, thus, M is a GK-manifold of a pointlike constant type c.

Thus the following theorem is proved.

THEOREM 4. *A GK-manifold is a manifold of pointlike constant type c then and only then, when the following is realized*

$$\mathcal{C}(X,Y,Z,W)=\langle\langle C(X,Y),C(Z,W)\rangle\rangle\, c\{\langle\langle W,Y\rangle\rangle\,\langle\langle Z,X\rangle\rangle - \\ -\langle\langle W,X\rangle\rangle\,\langle\langle Z,Y\rangle\rangle\}.$$

We introduce the following theorem giving the structural tensor properties.

THEOREM 5. *GK-structure structural tensors have the following properties:*

$$\begin{gathered} 1)\ \Phi\circ C(X,Y)=-C(\Phi X,Y)=-C(X,\Phi Y);\\ 2)\ \Phi\circ F=-F\circ\Phi;\\ 3)\ \langle\langle C(X,Y),Z\rangle\rangle+\overline{\langle\langle Y,C(X,Z)\rangle\rangle}=0;\\ 4)\ F(\xi)=0;\\ 5)\ \eta\circ F=0. \end{gathered} \tag{29}$$

Proof. 1) After covariant differentiation of the equality $\Phi^2=-id+\eta\otimes\xi$, we get $\nabla_Y(\Phi)\Phi X+\Phi\circ\nabla_{\Phi Y}(\Phi)X=\xi\nabla_Y(\eta)X+\eta(X)\nabla_Y\xi$. In the last equality we change $X\to\Phi X$, and the received identity will be influenced by operator Φ^2, then we get $\Phi\circ\nabla_Y(\Phi)\Phi X=\Phi^2\circ\nabla_Y(\Phi)\Phi^2X$. In the received identity we change $Y\to\Phi Y$, then

$$\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X=\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi^2X;\forall X,Y\in\mathcal{X}(M). \tag{30}$$

In the received identity we change $X\to\Phi X$, then we get $\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi^2X=-\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi X$; $\forall X,Y\in\mathcal{X}(M)$. Considering the last identity, from (19) we have

$$\Phi\circ C(X,Y)=-\frac{1}{2}\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi X=-\frac{1}{2}\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi^2X=-C(\Phi X,Y)$$

and

$$\begin{gathered} C(\Phi X,\Phi)=-\tfrac{1}{2}\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi^2X=\tfrac{1}{2}\Phi\circ\nabla_{\Phi^2X}(\Phi)\Phi Y=\\ =\tfrac{1}{2}\Phi^2\circ\nabla_{\Phi^2X}(\Phi)\Phi^2Y=-\tfrac{1}{2}\Phi^2\circ\nabla_{\Phi^2Y}(\Phi)\Phi^2X=\\ =-\tfrac{1}{2}\Phi\circ\nabla_{\Phi^2Y}(\Phi)\Phi X=C(X,\Phi Y). \end{gathered}$$

2) From the analytic expression of the second structural tensor and from the expression $F(X)=-\Phi^2\circ\nabla_{\Phi X}(\Phi)\xi-\Phi^2X=-\Phi\circ\nabla_{\Phi X}(\Phi)\xi-\Phi^2X$; $\forall X,Y\in\mathcal{X}(M)$, it follows that $\Phi\circ F=-F\circ\Phi$.

3) Considering (18) and (21) we have

$$\begin{gathered} \langle\langle C(X,Y),Z\rangle\rangle=\langle C(X,Y),Z\rangle+\sqrt{-1}\,\langle C(X,Y),\Phi Z\rangle=\\ =\left\langle-\tfrac{1}{2}\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X,Z\right\rangle+\sqrt{-1}\left\langle-\tfrac{1}{2}\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi X,\Phi Z\right\rangle=\\ =-\tfrac{1}{2}\,\langle\Phi X,\nabla_{\Phi Y}(\Phi)\Phi Z\rangle+\tfrac{1}{2}\sqrt{-1}\left\langle\Phi X,\nabla_{\Phi Y}(\Phi)\Phi^2Z\right\rangle=\\ =\tfrac{1}{2}\,\langle X,\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi Z\rangle-\tfrac{1}{2}\sqrt{-1}\left\langle X,\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi^2Z\right\rangle=\\ =\tfrac{1}{2}\,\langle X,\Phi\circ\nabla_{\Phi Y}(\Phi)\Phi Z\rangle-\tfrac{1}{2}\sqrt{-1}\left\langle X,\Phi^2\circ\nabla_{\Phi Y}(\Phi)\Phi Z\right\rangle=\\ =-\langle X,C(Y,Z)\rangle-\sqrt{-1}\,\langle X,\Phi C(Y,Z)\rangle=-\overline{\langle\langle X,C(Y,Z)\rangle\rangle}. \end{gathered}$$

4) Because $\Phi(\xi)=0$, we have $F(\xi)=-\Phi\circ\nabla_{\Phi\xi}(\Phi)\xi-\Phi^2\xi=0$.

5) Because the almost contact metrical manifold has equalities $\eta\circ\Phi=0$ and $\eta(\nabla_X\xi)=0$, then $\eta(F(X))=-\eta(\nabla_X\xi)-\eta(\Phi^2X)=0$, h.e. $\eta\circ F=0$.

Let us find the representation of the equality (28) of the adjoint G-structure. We fix the point $p \in M$, orthonormal frame $r = (p, e_1, ..., e_n)$ of space $T_p(M)$, that is considered as a $\mathbf{C}$-module, and the corresponding A-frame

$$r = (p, \epsilon_0, \epsilon_1, ..., \epsilon_n, \epsilon_{\hat{1}}, ..., \epsilon_{\hat{n}}),$$

where $\epsilon_a = \sqrt{2}\sigma_p(e_a)$, $\epsilon_{\hat{a}} = \sqrt{2}\bar{\sigma}_p(e_a)$, $\epsilon_0 = \xi_p$. Putting into (28) $X = e_a$, $Y = e_b$, $Z = e_c$, $W = e_d$, we get an equivalent (in point p) identity

$$\mathcal{C}(e_a, e_b, e_c, e_d) = \langle\langle C(e_a, e_b), C(e_c, e_d)\rangle\rangle = \\ = c\{\langle\langle e_d, e_b\rangle\rangle \langle\langle e_c, e_a\rangle\rangle - \langle\langle e_d, e_a\rangle\rangle \langle\langle e_c, e_b\rangle\rangle\}. \tag{31}$$

Because

$$\langle\langle e_a, e_b\rangle\rangle = \langle \sigma e_a, \sigma e_b\rangle + \sqrt{-1}\,\langle \sigma e_a, \sigma e_b\rangle = 2\,\langle \sigma e_a, \bar{\sigma} e_b\rangle = \left\langle \epsilon_a, \epsilon^b \right\rangle = \delta_a^b,$$

which also comes from the orthonormal nature of the frame r. Because $C^{abc} = \frac{1}{2}C(\epsilon_{\hat{b}}, \epsilon_{\hat{c}})^a$, then

$$\langle\langle C(e_a, e_b), C(e_c, e_d)\rangle\rangle = \langle C(e_a, e_b), C(e_c, e_d)\rangle + \sqrt{-1}\,\langle C(e_a, e_b), C(e_c, e_d)\rangle = \\ = 2\,\langle \sigma C(e_a, e_b), \bar{\sigma} C(e_c, e_d)\rangle = 2\,\langle C(\bar{\sigma} e_a, \bar{\sigma} e_b), C(\sigma e_c, \sigma e_d)\rangle = \\ = \tfrac{1}{2}\left\langle C(\epsilon_{\hat{a}}, \epsilon_{\hat{b}}), C(\epsilon_c, \epsilon_d)\right\rangle = 2\left\langle C^{hab}\epsilon_h, C_{gcd}\epsilon^g\right\rangle = \\ = 2C^{hab}C_{gcd}\,\langle \epsilon_h, \epsilon^g\rangle = 2C^{hab}C_{gcd} = 2C^{abh}C_{hcd}.$$

Then relations (31) can be formulated as $C^{abh}C_{hcd} = \frac{c}{2}\delta_{cd}^{ab}$, where $\delta_{cd}^{ab} = \delta_c^a\delta_d^b - \delta_c^d\delta_b^a$ is a Kronecker delta of the second order. Through this we prove the following theorem.

Theorem 6. *Let M be a GK-manifold. Then the following statements are equivalent:*
1) M is a GK-manifold of a pointlike constant type c.
2) The first structural tensor of a GK-manifold satisfies the identity

$$\langle\langle C(X, Y), C(Z, W)\rangle\rangle = c\{\langle\langle W, Y\rangle\rangle \langle\langle Z, X\rangle\rangle - \langle\langle W, X\rangle\rangle \langle\langle Z, Y\rangle\rangle\}.$$

3) On the adjoint G-structure space the following relation is correct

$$C^{abh}C_{hcd} = \frac{c}{2}\delta_c d^a b. \tag{32}$$

Let us differentiate externally the following equality (32)

$$dC^{abh}C_{hcd} + C^{abh}dC_{hcd} = 2dc\delta_{cd}^{ab}.$$

Considering the structural equations of GK-manifolds we have

$$(-C^{gbh}\theta_g^a - C^{agh}\theta_g^b - C^{abg}\theta_g^h + C^{abhg}\omega_g - 2\delta_g^{[a}F^{bh]}\omega^g - \\ -C^{abh}\omega)C_{hcd} + C^{abh}(C_{gcd}\theta_h^g + C_{hgd}\theta_c^g + C_{hcg}\theta_d^g + C_{hcdg}\omega^g - \\ -2\delta_{[h}^g F_{cd]}\omega_g - C_{hcd}\omega) = 2\delta_{cd}^{ab}dc.$$

Opening the brackets and collecting similar terms considering (32), (12_{10}), (12_{14}), (11), we have

$$(C^{abh}C_{hcgd} - \tfrac{2}{3}F^{ab}C_{gcd})\omega^g - (C^{ahgb}C_{hcd} + \tfrac{2}{3}C^{abg}F_{cd})\omega_g - 4c\delta_{cd}^{ab} = \\ = 2\delta_{cd}^{ab}(c_g\omega^g + c^g\omega_g + c_0\omega).$$

From here we have

$$1)\ \delta_{cd}^{ab}c_g = C^{abh}C_{hcgd} - \tfrac{2}{3}F^{ab}C_{gcd}; \\ 2)\ \delta_{cd}^{ab}c^g = -C^{ahgb}C_{hcd} - \tfrac{2}{3}F_{cd}C^{abg};\quad 3)\ c_0 = -2c. \tag{33}$$

Contracting equalities (33_1) and (33_2) at first by indices a and c, and then by indices b and d, considering the equalities $F_{ad}C^{dbc} = F^{ad}C_{dbc} = 0$ and $C^{abcg}C_{gdh} = C_{abcg}C^{gdh} = 0$, we get $n(n-1)c_g = n(n-1)c^g = 0$, h.e. either $dimM = 3$, or $c_g = c^g = 0$.

Thus $dc = -2c$. Differentiating the last equality externally we get $cd\omega = 0$, which, considering (10_1) will be written as: $c(F_{ab}\omega^a \wedge \omega^b + F^{ab}\omega_a \wedge \omega_b) = 0$, h.e. $cF_{ab}\omega^a \wedge \omega^b + cF^{ab}\omega_a \wedge \omega_b = 0$.

Therefore either $c = 0$, or $F_{ab} = F^{ab} = 0$.

Thus the following theorem is proved.

Theorem 7. *AGK-manifold of a constant non-zero type is a SGK-manifold of the II kind.*

Assume that M is a SGK-manifold of the II kind then we will perform a complete convolution (23): $\sum_{a,b,c} |C^a bc|^2 = C^{abc}C_{abc} = 2cn(n-1)$, where n is a complex dimensionality of the contact distribution L. This implies that $c \geq 0$, $c \in R$, at that $c = 0$ then and only then, when $C^{abc} = 0$, h.e. M is a Kenmotsu manifold. Thus it's proved that

Theorem 8. *The class of SGK-manifolds of the II kind of the zero constant type coincides with the class of Kenmotsu manifolds.*

We have to investigate SGK-manifolds of the II kind of the non-zero constant type. According to [4] SGK-manifolds of the II kind of the non-zero constant type are concircularly transformed into most precise cosymplectic manifolds which, in turn, are locally equivalent to the product of own (h.e. non-Keller) almost Keller eigenmanifold and the real straight line. Because the class of almost Keller manifolds of the non-zero constant type coincides with the class of six-dimensional almost Keller eigenmanifolds ([3], [14]), we can formulate the following theorem.

Theorem 9. *The class of SGK-manifolds of the II kind of the zero constant type coincides with the class of Kenmotsu manifolds. The class of SGK-manifolds of the II kind of the non-zero constant type is concircularly transformed into the almost contact metrical manifolds locally equivalent to the product of the six-dimensional NK-eigenmanifold and the real straight line.*

Theorems 8 and 9 can be combined into the following Fundamental theorem.

Theorem 10. ***Fundamental theorem.*** *The class of GK-manifolds of the zero constant type coincides with the class of Kenmotsu manifolds. The class of GK-manifolds of the non-zero constant type is concircularly transformed into the almost contact metrical manifolds locally equivalent to the product of the six-dimensional NK-eigenmanifold and the real straight line.*

6. Conclusions

In this work fully research generalized Kenmotsu manifolds constancy of type. The local characteristic of these manifolds is obtained. The main theorem gives a complete solution to the assigned task.

СПИСОК ЦИТИРОВАННОЙ ЛИТЕРАТУРЫ

1. Кириченко, В. Ф. Почти эрмитовы многообразия постоянного типа / В. Ф. Кириченко // Докл. АН СССР. – 1981. – Т. 256, № 6. – С. 1293–1297.

2. Gray, A. Curvature identities for Hermitian and almost Hermitian manifolds / A. Gray // Tôhoku Math. J. – 1976. – Vol. 28. – P. 601–612.

3. Кириченко, В. Ф. К-пространства постоянного типа / В. Ф. Кириченко // Сибирский Математический журнал. – 1976. – Т. 17, № 2. – С. 282–289.

4. Umnova, S. V. On conformal invariants of Kenmotsu manifolds / S. V. Umnova // Webs and Quasigroups, Tver State Univ. – 2002. – P. 155–160

5. Kenmotsy, K. A class of almost contact Riemannian manifolds / K. Kenmotsu // Tôhoku Math. J. 1972. – Vol. 24. – P. 93–103.

6. Blair, D. E. Almost contact manifolds with Killing structure tensors / D. E. Blair, D. K. Showders // J. Differential Geom. – 1974. – vol. 9. – P. 577–582.

7. Кириченко, В. Ф. Дифференциально-геометрические структуры на многообразиях / В. Ф. Кириченко. – Одесса: Печатный Дом, 2013. – 458 с.

8. Кириченко, В. Ф. Дифференциальная геометрия квази-сасакиевых многообразий / В. Ф. Кириченко, А. Р. Рустанов // Математический сборник. — 2002. – Т. 193, № 8. – С. 71–100.

9. Abu-Saleem Ahmad. Some aspects geometry of generalized manifolds Kenmotsu / Abu-Saleem Ahmad, A. R. Rustanov // Far East Journal of Mathematical Sciences (FJMS). – 2018. – vol. 103, no. 9. – pp. 1407–1432.

10. Абу-Салеем Ахмад. Свойства интегрируемости обобщенных многообразий Кенмоцу / Абу-Салеем Ахмад, А. Р. Рустанов, С. В. Харитонова // Владикавказский математический журнал. – 2018, т. 20, вып. 3. – С. 4–20.

11. Kirichenko, V. F. Generalized quasi-Kaehlerian manifolds and axioms of CR-submanifolds in generalized Hermitian geometry / V. F. Kirichenko // Geom. Dedicata. – 1994. – vol. 51. – pp. 75–104.

12. Арсеньева, О. Е. Q-алгебры основных типов почти эрмитовых и почти контактных метрических многообразий / О. Е. Арсеньева, В. Ф. Кириченко // Proc. Intern. Geom. Center. – 2013. – т. 6(4). – р. 34–43.

13. Кириченко, В. Ф. Аксиома голоморфных плоскостей в обобщенной эрмитовой геометрии / В. Ф. Кириченко // Доклады АН СССР. – 1981. – т. 260, № 4. – С. 795–799.

14. Кириченко, В. Ф. К-алгебры и К-пространства постоянного типа с индефинитной метрикой / В. Ф. Кириченко // Мат. заметки. – 1981. – т. 29, № 2. – С. 265–278.

15. Abu-Saleem Ahmad. Curvature Identities Special Generalized Manifolds Kenmotsu Second Kind / Abu-Saleem Ahmad, A. R. Rustanov // Malaysian Journal of Mathematical Sciences. – 2015. – vol. 9(12), pp. 187–207.

REFERENCES

1. Kirichenko, V. F., 1981, "Almost Hermitian manifolds of the constant type", *Report. ASUSSR*, vol. 256, no. 16, pp. 1293–1297.

2. Gray, A., 1976, "Curvature identities for Hermitian and almost Hermitian manifolds", *Tôhoku Math. J.*, vol. 28, pp. 601–612.

3. Kirichenko, V. F. 1976, "Constant type K-spaces", *Sib. Math. J.*, vol. 17, no. 12, pp. 282–289.

4. Umnova, S. V., 2002, "On conformal invariants of Kenmotsu manifolds", *Webs and Quasigroups*, Tver State Univ., pp. 155–160.

5. Kenmotsy, K., 1972, “A class of almost contact Riemannian manifolds“, *Tôhoku Math. J.*, vol. 24, pp. 93–103.

6. Blair, D. E., Showders, D. K., 1974, “Almost contact manifolds with Killing structure tensors“, *J. Differential Geom.*, vol. 9, pp. 577–582.

7. Kirichenko, V. F., 2013, *Differential geometric structures on manifolds*, Petchatny Dom, Odessa, 458 p.

8. Kirichenko, V. F., Rustanov, A. R., 2002, “Quasi-Sasakian manifolds differential“, *Mathematical collection*, vol. 193(18), pp. 71–100.

9. Abu-Saleem Ahmad, Rustanov, A. R., 2018, “Some aspects geometry of generalized manifolds Kenmotsu“, *Far East Journal of Mathematical Sciences (FJMS)*, vol. 103, no. 9, pp. 1407–1432.

10. Abu-Saleem, A., Rustanov, A. R., Kharitonova S. V., 2018, “Generalized Kenmotsu manifold integrability properties“, *Vladikavkasian Mathematical Journal*, vol. 20(3), pp. 4–20. DOI 10.23671/VNC.2018.3.13829.

11. Kirichenko, V. F.,1994, “Generalized quasi-Kaehlerian manifolds and axioms of CR-submanifolds in generalized Hermitian geometry“, *Geom. Dedicata*, vol. 51. pp. 75–104.

12. Arsenyeva, O. E., Kirichenko, V. F., 2013, “Q-algebras of the main types of almost Hermitian and almost contact metrical manifolds“, *Proc. Intern. Geom. Center*, vol. 6, no. 4, pp. 34–43.

13. Kirichenko, V. F., 1981, “Holomorphic plane axiom in generalized Hermitian geometry“, *ASUSSR reports*, vol. 260(14), pp. 795–799.

14. Kirichenko, V. F., 1981, “K-algebras and K-spaces of the constant type with indefinite metrics“, *Math. notes*, vol. 29(12), pp. 265–278.

15. Abu-Saleem, A., Rustanov, A. R., 2015, “Curvature Identities Special Generalized Manifolds Kenmotsu Second Kind“, *Malaysian Journal of Mathematical Sciences*, vol. 9(12), pp. 187–207.

Получено 6.03.2019 г.
Принято в печать 12.07.2019 г.