ЧЕБЫШЕВСКИЙ СБОРНИК
Том 26. Выпуск 3.

УДК 517 DOI 10.22405/2226-8383-2025-26-3-274-283

Индекс Альбертсона и индекс Сигмы в деревьях, заданных последовательностями степеней

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Аннотация

В нашей статье мы изучаем индекс Альбертсона и сигма-индекс в деревьях, заданных степенными последовательностями, и ввели нерегулярность индекса Альбертсона и сигма-индекса для последовательностей степени(2,3,4), подкрепив это набором иллюстративных примеров получения общего соотношения в виде:

$$\operatorname{irr}(G) = (d_n - 1)^2 + (d_{n-1} - 1)^2 + \sum_{i=1}^{n-2} (d_i - 1)(d_i - 2) + d; \text{ d:известное значение.}$$

Ключевые слова: Дерево, индекс Альбертсона, индекс Сигмы, степень, последовательность.

Библиография: 15 названий.

Для цитирования:
Хамуд, Ж., Абдулла, Д. Индекс Альбертсона и индекс Сигмы в деревьях, заданных последовательностями степеней // Чебышевский сборник, 2025, т. 26, вып. 3, с. 274–283.

CHEBYSHEVSKII SBORNIK
Vol. 26. No. 3.

UDC 517 DOI 10.22405/2226-8383-2025-26-3-274-283

Albertson index and Sigma index in trees given by degree sequences

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Abstract

In our paper we study Albertson Index and Sigma index in Trees Given by Degree Sequences and introduced the irregularity of Albertson Index and Sigma index for sequences of degree(2,3,4), We supported this with a set of illustrative examples of obtaining the general relationship as:

$$\operatorname{irr}(G) = (d_n - 1)^2 + (d_{n-1} - 1)^2 + \sum_{i=1}^{n-2} (d_i - 1)(d_i - 2) + d;\ \text{d:known value.}$$

Keywords: Tree, Albertson Index, Sigma index, Degree, Sequence.

Bibliography: 15 titles.

For citation:
Hamoud, J., Abdullah, D. 2025, "Albertson index and Sigma index in trees given by degree sequences", *Chebyshevskii sbornik*, vol. 26, no. 3, pp. 274–283.

1. Introduction

Throughout this paper[1], Let $G = (V, E)$ be a simple graph with V vertices and E edges. Let Δ and δ indicate, respectively, the minimum and maximum degrees of vertices in $V(G)$ are had provided by Lina, Z., et al. in [1] and in [2]. The vertex set of a graph G is typically denoted as $V(G) = \{v_1, v_2, \ldots, v_n\}$ and the edge set of a graph G is typically denoted as $E(G) = \{e_1, e_2, \ldots, e_m\}$. We say that both vertices v_i and v_j are adjacent if there is an edge between them, and this edge is denoted as $\{v_i, v_j\}$. Additionally, vertex v_i is incident to edge $\{v_i, v_j\}$. The open neighborhood [2] of a vertex v, denoted $N(v)$, is defined as the set of all vertices adjacent to v within the graph, formally expressed as $N(v) = \{w \in V : \{v, w\} \in E\}$, where V is the vertex set and E is the edge set. The closed neighborhood of v, denoted $N[v]$, encompasses both the open neighborhood and the vertex itself, given by $N[v] = N(v) \cup \{v\}$. Let $\deg_G(v)$ be the degree of a vertex v is the number of edges incident with v or, equivalently, $\deg(v) = |N(v)|$.

Let $e = uv \in E(G)$ an edge, then the imbalance of e known as $|d_C(u) - d_G(v)|$ it is employee in "Albertson Index". Dorjsembe,S., et al. in [3] for a path $u_0u_1\cdots u_t$ in graph G, then $imb_G(u_0, u_t) = \sum_{i=0}^{t-1} |d_G(u_i) - d_G(u_{i+1})|$. In 1997, Albertson in [15] mention to the imbalance of an edge uv by $imb(uv)$ where we considered a graph G is regular if all of its vertices have the exact same degree, then the irregularity measure defined in [15, 16, 17] as:

$$\operatorname{irr}(G) = \sum_{uv \in E(G)} |d_u(G) - d_v(G)|. \tag{1}$$

Ali, A. et al in [4] mention to total irregularity of $\mathscr{D}(G) = (d_1, d_2, \ldots, d_i)$ a degree sequence of G where $d_1 \geqslant d_2 \geqslant \cdots \geqslant d_n$, it defined in Definition 1, then we have:

$$\operatorname{irr}(G) = 2(n+1)m - 2\sum_{i=1}^{n} id_i. \tag{2}$$

Thus, a "total irregularity of Albertson index", defined in [5, 1, 4] as:

$$\operatorname{irr}_T(G) = \sum_{|u,v| \leq V(G)} |d_G(u) - d_G(v)|, \quad \operatorname{irr}_T(G) = \sum_{\{u,v\} \in E(G)} |d_G(v) - d_G(u)|.$$

[1]The paper was supported by the Russian Science Foundation under grant no. 22-11-00177.

The recently introduced $\sigma(G)$ irregularity index is a simple diversification of the previously established Albertson irregularity index, in [7, 8] defined as:

$$\sigma(G)=\sum_{uv\in E(G)}\left(d_u(G)-d_v(G)\right)^2.$$

The sigma index in graph theory refers to a measure of the degree differences between vertices in a graph. The goal is to determine the graphs that have the maximum sigma index within certain classes of graphs. Abdo, Dimitrov, and Gutman characterized the graphs with the greatest sigma index (see [9, 5, 10, 11, 12, 13, 14]) among all connected graphs of a fixed order as:

1. If and only if G is regular, then that's mean: $d_G(u)=d_G(v);\forall uv\in E(G)$ holds

$$\sigma(G)=\operatorname{irr}(G)=\sum_{uv\in E(G)}\left|d_G(u)-d_G(v)\right|=0 \tag{3}$$

2. If and only if G is stepwise regular, then that's mean: $d_G(u)>d_G(v);\forall uv\in E(G)$ or $d_G(v)>d_G(u);\forall uv\in E(G)$. Then

$$\sigma(G)=\operatorname{irr}(G)=\sum_{uv\in E(G)}\left|d_G(u)-d_G(v)\right|>0 \tag{4}$$

Hypothesize 1. *let be a sequences $(d_n,\ldots,d_i,\ldots,\ \mathrm{d}_{n-1})$ with $n\geq 0$ and let be order as: $d_n>d_{n-i}>\ldots>d_1>d_2>\ldots$ so that we have a relation as:*

$$\operatorname{irr}(G)=(d_n-1)^2+(d_{n-1}-1)^2+\sum_{i=1}^{n-2}(d_i-1)(d_i-2)+d;d:\ \textit{known value}$$

This paper is organized as follows. In Section 1, we observe the important concepts to our work including literature view of most related papers, provided a preface through some of the important theories and properties, in Section 2 we introduced the main result of our paper and the goal of this paper according to Sigma index with degree sequence of order at least 4.

2. Main Result

Definition 1 (Degree Sequence [6]). *Let $G=(V,E)$ be a simple graph, where $V=\{v_1,v_2,\ldots,v_n\}$, let $\mathscr{D}(G)=(d_G(v_1),d_G(v_2),\ldots,d_G(v_n))$ be a degree sequence of G where $d_G(v_1)\geqslant d_G(v_2)\geqslant\geqslant\cdots\geqslant d_G(v_n)$. When $\mathscr{D}(G)=(k,k,\ldots,k)$, then G is regular of degree k. Otherwise, the graph is irregular*

Лемма 1. *let $\mathscr{D}=(d_1,d_2,d_3)$ be a degree sequence, where $d_1\geq d_2\geq d_3$ of order n where $n\leq 4$ we have: $irr_{\ max}-irr_{\ min}=2(d_2-d_3)$ that is mean: $d(\ irr_{\ max},\ irr_{\ min})=2(d_2-d_3)$ if $d_2=d_3\Longrightarrow d_G(\ irr_{\ max},\ irr_{\ min})=2$, where The distance $d_G(u,v)$ between two vertices u,v of G is the length of one of the shortest (u,v)-path in G.*

Proposition 1. *Let $\mathscr{D}=(d_1,d_2,d_3)$ a degree sequence where $d_1\geqslant d_2\geqslant d_3$, then Albertson index define as:*

$$\operatorname{irr}(T)=\begin{cases}\operatorname{irr}_{max}(T)=(d_1-1)^2+(d_2-1)^2+(d_3-1)(d_3-2)(d_1-d_3)(d_2-d_3)\\ \operatorname{irr}_{min}(T)=(d_1-1)^2+(d_3-1)^2+(d_2-1)(d_2-2)+(d_1-d_3).\end{cases}$$

We have 3 cases shown in Table 1 as:

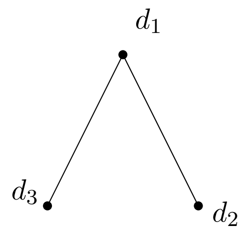
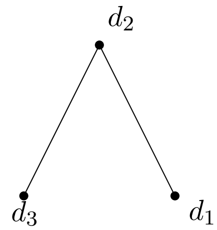
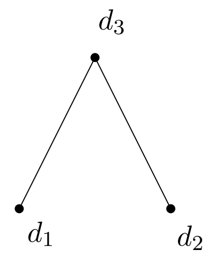
case 1	case 2	case 3
		

Таблица 1: positive integer d_1, d_2, d_3.

we have for Albertson Index irregularity : Case 1 = Case 2 $\leq$ Case 3.

Доказательство. We will discuss the following cases:

Case 1: In this case, we have:

$$\begin{aligned}\operatorname{irr}(T) &= (d_3-1)^2+(d_2-1)^2+(d_1-2)(d_1-1)+(d_1-d_3)+(d_1-d_2)\\ &= (d_3-1)^2+(d_2-1)^2+(d_1-1)^2+(d_1-d_2-d_3+1)\\ &= \sum_{i=1}^{3}(d_i-1)^2-\sum_{i=1}^{3}d_i+1.\end{aligned}$$

Case 2: In this case, we have also:

$$\begin{aligned}\operatorname{irr}(T) &= (d_3-1)^2+(d_1-1)^2+(d_2-2)(d_2-1)+(d_2-d_3)+(d_1-d_2)\\ &= (d_3-1)^2+(d_1-1)^2+(d_2-1)^2+(d_1-d_2-d_3+1)\\ &= \sum_{i=1}^{3}(d_i-1)^2-\sum_{i=1}^{3}d_i+1.\end{aligned}$$

Case 3: In this case, we have:

$$\begin{aligned}\operatorname{irr}(T) &= (d_1-1)^2+(d_2-1)^2+(d_3-2)(d_3-1)+(d_1-d_3)+(d_2-d_3)\\ &= (d_1-1)^2+(d_2-1)^2+(d_3-1)^2+d_1+d_2-3d_3+1\\ &= \sum_{i=1}^{3}(d_i-1)^2+d_1+d_2-3d_3+1.\end{aligned}$$

From Case 2, Case 3, we found:

$$\begin{aligned}\operatorname{irr}(T) &= d_1+d_2-3d_3+1-d_1+d_2+d_3-1\\ &= 2d_2-2d_3 \geq 0.\end{aligned}$$

Now we observe that, The first and Second cases are equivalent and their relationship with the third case is expressed as follows: Case 1 = Case 2 $\leq$ Case 3. As desire. $\square$

Remark 1. *Let a sequence $d = (d_1, d_2, d_3)$ where $d_1 \geq d_2 \geq d_3$ and we will define a sequences is: $d = (4, 3, 2)$ we have 3 cases we show that in Table 2 according to Proposition 1 for both Albertson index and Sigma index according to definition of both indices:*

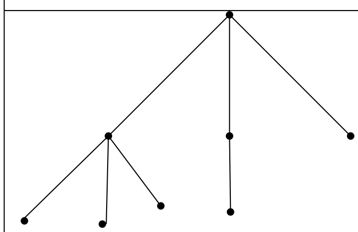
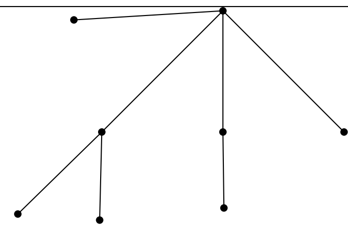
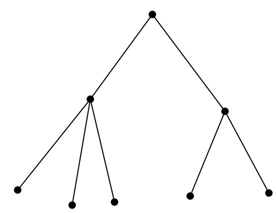
Case 1	**Case 2**	**Case 3**
		
irr = 14	irr = 14	irr = 16
$\sigma = 34$	$\sigma = 32$	$\sigma = 40$

Таблица 2: Albertson Index and Sigma Index for a sequence d_1, d_2, d_3.

Remark 2. *Let a sequence $d = (d_1, d_2, d_3, d_4)$ where $d_1 \geq d_2 \geq d_3 \geq d_4$ to improving Example* 1 and we will define sequences is: $d = (8, 5, 4, 2)$ in Figure 1 we have 24 distinct ways, for example shown in Figure 1, we can discuss many cases according to the following Figure 1 where we introduce that according to Proposition 1 as we show that:

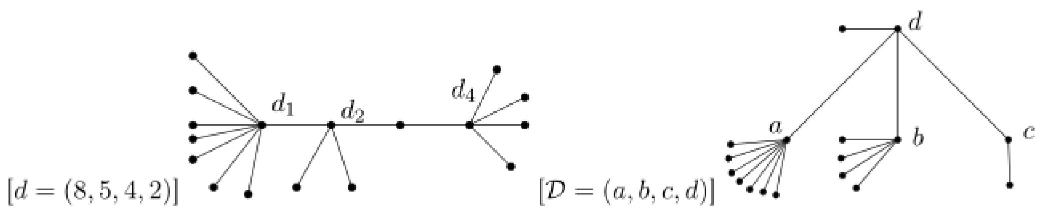


Рис. 1: Sequence define as $\mathcal{D} = (a, b, c, d)$.

if $d = 2$ we have:
$49 + 16 + 4 + 2 + 9 = 80 \Rightarrow$ irr = 80
$343 + 16 + 27 + 84 = 470 \Rightarrow \sigma = 470$
From this case we can formed many methods to obtain on maximum value, we will take for example $d = 4$ we have;

$$3 + 1 + 16 + 49 + 2 + 1 + 4 = 76 \Rightarrow \text{irr} = 76$$
$$9 + 22 + 343 + 64438 \Rightarrow \sigma = 438$$

We can formulate that as:
$\text{irr} = (a-1)^2 + (b-1)^2 + (c-1)^2 + (d-a) + (d-b) + (d-c) + (d-1)(d-3)$,
$if(d = 8, a = 5, b = 4, c = 2) \Rightarrow \text{irr} = 74$

If $d = 5$ we have:
$8 + 10 + 49 + 10 = 74 \Rightarrow \text{irr} = 74$
$32 + 19 + 343 + 28 = 422 \Rightarrow \sigma = 422$

If $d = 8$ we have:
$35 + 10 + 29 = 74 \Rightarrow \text{irr} = 74$
$64 + 27 + 36 + 26 + 245 = 398 \Rightarrow \sigma = 398$

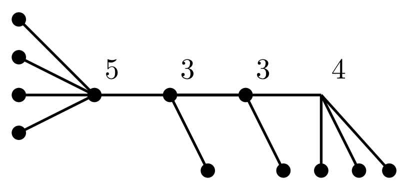
we will get on:

sequence of d	irr	σ
2	80	470
4	76	438
5	74	422
8	74	398

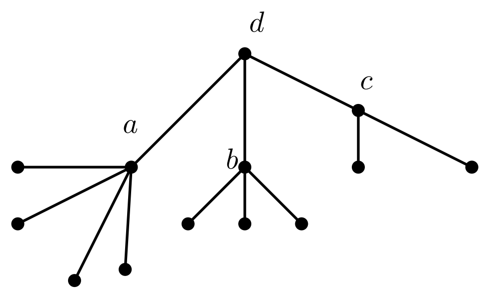
In the end we get on:

irr		σ	
max	min	max	min
80	74	470	398

Remark 3. *let a sequence* $d_1, d_2, d_3, d_4; d_1 \geqslant d_2 \geqslant d_3 \geqslant d_4$ *and we will suppose sequences is:* $(5,4,3,3)$ *we have 12 options, for example we take: Let we take:*



We can discuss many cases according to the following:



if $d = 3$ *we have:*
$16 + 11 + 5 = 32 \Rightarrow irr = 32$
$64 + 32 + 8 = 84 \Rightarrow \sigma = 84$
We can formulate that as:
$irr = (a-1)^2 + (b-1)^2 + (c-1)^2 + (d-a) + (d-b) + (d-c) + (d-1)(d-3)\,,$
$if(d = 5, a = 4, b = 3, c = 3) \Rightarrow irr = 30$

if $d = 4$ *we have:*
$16 + 10 + 4 = 30 \Rightarrow irr = 30$
$64 + 10 + 18 = 92 \Rightarrow \sigma = 92$

if $d = 5$ *we have:*
$10 + 20 = 30 \Rightarrow irr = 30$
$28 + 32 + 24 = 84 \Rightarrow \sigma = 84$

In the end we get on:

irr		σ	
max	min	max	min
32	30	92	84

Hypothesize 2. *Let T be tree of order $n = 4$, a degree sequence $\mathscr{D} = (d_1, d_2, d_3, d_4)$ where $d_1 \geqslant d_2 \geqslant d_3 \geqslant d_4$. Then we have:*

$$\operatorname{irr}(T) = \sum_{i=1}^{3}(d_i - 1)^2 + \sum_{i=1}^{3}(d_4 - d_i) + (d_4 - 1)(d_4 - 3).$$

Доказательство. Let be consider sequence $\tilde{d} = (d_1, d_2, d_3, d_4)$ with the term $d_4 > d_1 \geq d_2 \geq d_3$, so that according to Proposition 1 for $d_1 \geq d_2 \geq d_3$ we have $\sum_{i=1}^{3}(d_i - 1)^2$ it is known the sum of squared deviations of three values of vertices d_1, d_2, d_3 where $d_1 \geq d_2 \geq d_3$. Thus $\sum_{i=1}^{3}(d_i - 1)^2 = \sum_{i=1}^{3} d_i^2 - 2\sum_{i=1}^{3} d_i + 3$.

According to our term $d_4 > d_1 \geq d_2 \geq d_3$ we notice for vertices d_4 it is define $d_4^2 - 4d_4 + 3 = (d_4 - 1)(d_4 - 3)$ true occurs only when $d_4 \geq 3$ as we know topological indices quantifying structural irregularity define the sum of degree is $2(n-1)$, So that when $d_4 > d_1 \geq d_2 \geq d_3$ then we have $d_4 - d_1$ and $d_4 - d_2$ and $d_4 - d_3$ we can express that by $\sum_{i=1}^{3}(d_4 - d_i)$.

In essence, this relationship is realised according to the fact that each of its terms under the given term is satisfied and therefore, according to the definition of the Albertson index, the relationship is valid according to the given condition. As desire. □ In the following examples, we will improvement the sequence to become for 4 and 6 vertices which sequential and order that have a path-like according to Proposition 1 and Hypothesize 2 as we show.

Remark 4. *Let be T tree with a sequence $d = (d_1, d_2, d_3, d_4)$ where $d_1 \geq d_2 \geq d_3 \geq d_4$ and we will define sequences as:* $(5, 5, 4, 4, 4, 3, 3, 2, 2)$. *Let be order sequences, we have 24 option, as we show more options such that:*

1. *the sequence is :* $(5, 4, 4, 3, 2, 2, 3, 4, 5)$. *In this option we have path- like tree: 6.*
2. *the sequence is :* $(5, 4, 3, 2, 2, 3, 4, 4, 5)$. *In this option we have path- like tree: 6.*

So that we can define Figure 2 according to improvement Example 2 as we show a path-like according to Proposition 1 and Hypothesize 2 as:

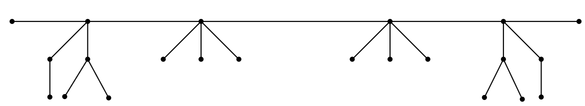


Рис. 2: Paths with Sequence $d = (d_1, d_2, d_3, d_4)$.

To determine Albertson index and Sigma index according to the term as

$$\begin{cases} \operatorname{irr}(T) = 66, \\ \sigma(T) = 140. \end{cases}$$

3. Conclusion

Through this paper, we have found the maximum and minimum value relationships corresponding to the trees with a sequential degree 2,3 and 4 , which in turn will lead us in future research to find the general relationship through which we can calculate the maximum and minimum values of all values and bounds.

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Получено: 03.06.2025
Принято в печать: 27.08.2025