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Расстояние $Wt-$ над метрическим пространством $b-$

Э. Альмухур, М. Кусини, А. Абир, А. Манал

Альмухур Эман — факультет фундаментальных и гуманитарных наук, Частный университет прикладных наук (г. Амман, Иордания).
email: e_almuhur@asu.edu.jo
Кусини Майсун — кафедра математики, Иорданский университет Аль-Зайтуна (г. Амман, Иордания).
email: M.qousini@zuj.edu.jo
Альнана Абир — факультет математики, Университет принца Саттама бин Абдулазиза (г. Аль-Хардж, Саудовская Аравия).
email: a.alnana@psau.edu.sa
Аль-Лабади Манал — кафедра математики, Университет Петры (г. Амман, Иордания).
email: Manal.allabadi@uop.edu.jo

Аннотация

В этой работе мы исследуем характеристики $wt-$расстояния характеристики над $b-$метрическим пространством и условия, необходимые для обеспечения наличие неподвижной точки, если позволить $\beta-$функции соответствующим образом. Кроме того, мы доказываем некоторые теоремы о неподвижной точке.

Ключевые слова: $wt-$ метрика, $b-$ метрика, $\beta-$ функция, неподвижная точка.

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$Wt-$ Distance over $b-$ Metric Space

E. Almukhur, M. Kusini, A. Alnana, M. Al-Labadi

Almukhur Eman — department of basic science and humanities, Applied Science Private University (Amman, Jordan).
e-mail: e_ almuhur@asu.edu.jo
Qousini Maysoon — department of mathematics, Al-Zaytoonah University of Jordan (Amman, Jordan).
e-mail: M.qousini@zuj.edu.jo
Alnana Abeer — department of mathematics, Prince Sattam Bin Abdulaziz University (Al-Kharj, Saudi Arabia).
e-mail: a.alnana@psau.edu.sa
Al-Labadi Manal — department of mathematics, University Of Petra (Amman, Jordan).
e-mail: Manal.allabadi@uop.edu.jo

Abstract

In this paper, we examine the $wt-$distance characteristics over $b-$metric space and the conditions required to ensure the presence of the fixed point by letting $\beta-$function appropriately. In addition, we prove some fixed point theorems.

Keywords: $wt-$ metric, $b-$ metric, $\beta-$ function, fixed point.

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1. Introduction

One of the first ideas that humans developed was the concept of distance. Distance was initially conceptualized by (Euclid). Felix Hausdorff later redefined "metric space"as the general form and more axiomatic version that was first discussed by Maurice René Frechét as "$L-$space."The concept of distance has since been explored, improved upon, and broadly applied in numerous contexts. In this paper, we concentrate on two of these generalizations: b-metric and wt-distance. We'll set up some notations and ideas before we begin to investigate the topic in depth. We assume that all sets and subsets examined in this paper are non-empty throughout. The function d defined on $X \times X$ to $\mathbb{R}^+$ is the distance function if the following axioms are satisfied for all u, v and ω in X :

(i) $d(u, v) = d(v, u)$.

(ii) $d(u, v) = 0$ if $u = v$.

(iii) $d(u, v) + d(v, \omega) \geq d(u, \omega)$ the triangle inequality as it states that the sum of a triangle's two sides is at least as large as the third side when applied to $\mathbb{R}^2$ with the usual metric.

A non-empty set X together with a function d is a metric space. We short (X, d) by X.

The space (X, d) is complete if none of its points are missed from its inside or boundary. For example, the sequence $(x_n)_{n=1}^{\infty}$ in the metric space X is complete if $\forall n \in \mathbb{N}$, $\exists k \in \mathbb{Z} : \forall x_1, x_2 > k$, $d(x_1, x_2) < n$.

On the other side, the set of rational numbers is not complete since we cannot construct a Cauchy sequence of rational numbers that converges to a rational number.

The hyperbolic metric space, introduced by Mikhael Gromov [3], is defined as: X is $d-$hyperbolic iff all $x, y, z, \omega \in X$ we have:

$\min((x, y)_\omega, (y, z)_\omega) - d \leq (x, z)_\omega$......(1)

If (1) is satisfied $\forall \omega_o$ a fixed base point and $\forall x, y, z \in X$, then it is satisfied for all with a constant $2d$.

A space (X, d) is called a pseudometric if $\forall x, y \in X$, for $x \neq y$, one may have $d(x, y) = 0$. This notion is introduced by Đuro Kurepa [4]. Typically, every metric space is pseudometric. The pseudometric topology is generated by open balls defined as $B_r(a) = \{x \in X : d(a, x) < r\}$.

A space (X, d) is said to be a $\nu-$generalized metric space [5] if $\forall\ x \neq y$ in X, we have the following:

i) $d(x, y) = 0$.

ii) $d(x, y) = d(y, x)$.

iii) $d(x, y) \leq d(x, z_1) + d(z_1, z_2) + ... + d(z_\nu, y)\ \forall z_1 \neq z_2 \neq ... \neq z_\nu \in X$.

If X is a non-empty set, then the partial metric is the function $p : X \times X \to \mathbb{R}$ such that $\forall a, b, c \in X$, the following conditions hold:

i) $a = b$ iff $p(a, a) = p(a, b) = p(b, b)$

ii) $p(a, a) = p(a, b)$

iii) $p(a, b) = p(b, a)$

iv) $p(a, b) \leq p(a, c) + p(z, b) - p(c, c)$

The partial metric space is the couple (X, p).

In 1998, Czerwik [1] and Bakhtin [2] introduced the extension $b-$metric space. The metric space (X, d) is a $b-$metric space over the constant k if the following hold $\forall x, y, z \in X$:

(i) $d(x, y) = d(y, x)$.

(ii) $d(x, y) = 0$ iff $x = y$.

(iii) If the relaxed triangle inequality hols for some constant $k \geq 1$:

$d(x, z) \leq k[d(x, y) + d(y, z)]$.

We see that any $b-$metric space is unquestionably a metric space under the scenario where $k = 1$. So, this idea is less strong than the concept of metric space.

2. $b-$MetricSpace

LEMMA 1. *If (X, d) is a $b-$metric space, then for then natural number n and $(x_0, x_1, ..., x_n) \in X^{n+1}$, we have*

$$d(x_0, x_n) \leq \textstyle\sum_{i=1}^{n-2} k^{i+1} d(x_i, x_{i+1}) + k^{n-1} d(x_{n-1}, x_n)......(2)$$

In Euclidean space, the convergence of a sequence $\{x_n\}_{n=1}^{\infty}$ to the point x is defined as [6]: if $\forall \epsilon > 0$, $\exists n \in \mathbb{N} : \forall n > N$, $d(x_n, x) < \epsilon$.

Such concept in topology is defined as: the sequence $\{x_n\}_{n=1}^{\infty}$ converges to the point x if U open set containing x, $\exists n \in \mathbb{N} : \forall n > N$, $x_n \in U$.

Both of these concepts are valid and equivalent in metric spaces.

DEFINITION 1. *For the sequence $\{x_n\}_{n=1}^{\infty}$ in the $b-$metric space (X, d) and a subset A in X [6]:*
(i) $\{x_n\}_{n=1}^{\infty}$ converges to x if $\lim_n d(x_n, x) = 0$.
(ii) $\{x_n\}_{n=1}^{\infty}$ is Cauchy if $\lim_n \sup\{d(x_n, x_m)\} = 0\ \forall m > n$.

(iii) $\{x_n\}_{n=1}^{\infty}$ *is complete if every Cauchy sequence converges.*
(iv) A is closed if for any convergent sequence $\{x_n\}_{n=1}^{\infty} \subset A$,
$\lim_{n\to\infty}(x_n) \in A$.
(v) A is bounded if $\sup\{d(x,y)\} < \infty \ \forall x, y \in A$.

Lemma 2. *The sequence* $\{x_n\}_{n=1}^{\infty}$ *in the* $b-$*metric space* (X,d) *is Cauchy [7] if* $\exists m \in \in [0, \frac{1}{k}] : d(x_{n+1}, x_{n+2}) \leq md(x_n, x_{n+1}) \ \forall n \in \mathbb{N}$.

Note that every $b-$metric space is metrizable, even though not all $\nu-$generalized metric spaces are metrizable. As a result, we observe that definition 2.2 above leaves no opportunity for ambiguity.

Let $CB(X) = \{F \subset X : F \neq \phi$ closed and bounded$\}$ and $\forall \ x \in X$, then $\forall A, B \subset X$, if $d(x, A) = \inf\{d(x, y) : y \in A\}$, then the Hausdorff metric space or Pompeiu–Hausdorff distance (H, d) [8] is defined by

$H(A, B) = \max\{\sup\{d(x, B) : x \in A\}, \sup\{d(y, A) : y \in B\}$.

Remark 1. *Define the function* $f : \mathbb{N} \to \mathbb{N} \cup \{0\}$ *by*
$f(n) = -[-\log_2 n]$......(3)
If $n \in \mathbb{N}$ *and* $(x_0, x_1, ..., x_n) \leq k^{f(n)} \sum_{i=0}^{n-1} d(x_i, x_{i+1})$ *and the following hold:*
(i) $f(2n) = f(n) + 1$
(ii) $f(n+1) \in \{f(n), f(n)+1\}$
(iii) f *is non-decreasing.*

Lemma 3. *If* $\{x_n\}_{n=1}^{\infty}$ *is a sequence in the* $b-$*metric space* (X,d), *and* $r \in [0,1)$ *:* $d(x_{n+1}, x_{n+2}) \leq r.d(x_n, x_{n+1}) \ \forall n \in \mathbb{N}$, *then* $\{x_n\}_{n=1}^{\infty}$ *is a Cauchy sequence [6].*

Доказательство. If $r = 0$, the result holds.
If $0 < r < 1$, then for some $s \in \mathbb{N}$: $kr^{2s} < 1$.
Define the function f be defined as (3).
For $n, m \in \mathbb{N} : m < n < m+2$, by remark 2.4 we have:

$$\begin{aligned} d(x_m, x_n) &< k^{f(n-m)}.\sum_{i=m}^{n-1} d(x_i, x_{i+1}) \\ &< k^s.\sum_{i=m}^{n-1} d(x_1, x_2) \\ &< k^s.\sum_{i=m}^{\infty} r^{i-1} d(x_1, x_n) \\ &< k^s r^m A \end{aligned}$$

where $A = \frac{d(x_1,x_2)}{r(1-r)}$
Now, $m + 2^{s<n}$ and $\nu = \left[\frac{n-m}{2^s}\right]$,so

$$\begin{aligned} d(x_m, x_n) &< \sum_{i=0}^{\nu} k^{i+1} d(x_{m+2^s i}, x_{m+(i+1)2^s}) + k^{\nu} d(x_{m+\nu 2^s}, x_n) \\ &< \sum_{i=0}^{\nu} k^{i+s+1}.r^{m+i2^s}.A + k^{\nu+s}.r^{m+v+2^s}.A \\ &< r^m.A \sum_{i=0}^{\nu} k^{i+s+1}.r^{m+i2^s} \\ &< r^m.A \sum_{i=m}^{\infty} k^{i+s+1}.r^{m+i2^s} \\ &< r^m A \frac{k^{s+1}}{1-kr^{2s}} \end{aligned}$$

Thus, $\{x_n\}_{n=1}^{\infty}$ is a Cauchy sequence. □

Теорема 1. *If* (X,d) *is a* $b-$*metric space and the function*
$g : \mathbb{N} \cup \{0\} \to [0, \infty)$ *defined as:*
$g(n) = 0$ *if* $n = 0$ *and* $g(n) = (2n - 2^{f(n)})k^{f(n)} + (2^{f(n)} - n)k^{f(n)-1} \ \forall n \in \mathbb{N}$
g *is strictly increasing.*
$g(n) = kg\left[\frac{n}{2}\right] + g(n - \left[\frac{n}{2}\right])$......(4)
$0 < g(n-1) - g(n-2) \leq g(n) - g(n-1)$......(5)
$g(n) \leq k(g(k) + g(n-k))$......(6)
$\forall k \in \mathbb{N}$ *and* $2 \leq n$ *where* $k < n$ *[6].*

Теорема 2. *If (X,d) is a complete $b-$metric space and $f : X \to CB(X)$, and $\exists r \in [0, \frac{1}{k}]$ such that $\forall x, y \in X$,*
$H(f(x), f(y)) \leq rd(x,y)$......(7)
we have the following:
(i) [1] $\exists z \in X : z \in z \in f(z)$.
(ii) [6] $\exists \epsilon > 0 : d(x,y) < \epsilon$.
(iii) [6] $\exists x \in X : d(x, f(x)) < \epsilon$.

Доказательство. If $p = \frac{1+r}{2} \in (0,1)$, then $\forall x, y \in X$ and $u \in f(x)$ with $d(x,y) < \epsilon$, $\exists \nu \in f(y)$: $d(u,\nu) \leq pd(x,y)$.
Then, if $\{u_n\}_{n=1}^{\infty} \in X : d(u_1, f(u_1)) \leq d(u_1, u_2) < \epsilon$
so, $u_{n+1} \in f(u_n)$ and $d(u_{n+1}, u_{n+2}) \leq pd(u_n, u_{n+1})$.
$\forall n \in \mathbb{N}$ and by lemma 2.3, $\{u_n\}_{n=1}^{\infty}$ is Cauchy.
But, X is complete, so $\{u_n\}_{n=1}^{\infty} \to z$ in X and
$d(z, f(z)) \leq \lim_{n\to\infty} k(d(z, u_{n+1}) + d(u_{n+1}, f(z)))$
$= k \lim_{n\to\infty} d(z, f(z))$
$\leq k \lim_{n\to\infty} H(f(u_n), f(z))$
$\leq kp \lim_{n\to\infty} d(u_n, z))$
$= 0$.
Hence, $f(z)$ is closed and $z \in f(z)$. □

Corollary 1. *If (X,d) is a complete $b-$metric space and*
$f : X \to CB(X)$. Define a bijective function $l : [0,\infty) \to [0,1)$ such that $\forall x, y \in X$ we have $H(f(x), f(y)) \leq l(d(x,y))d(x,y))$ and
$\lim_{\alpha\to t+0} \sup l(\alpha) < 1$ $\forall t \in [0,\infty)$, then $\exists z \in X : z \in f(z)$.

Доказательство. $\limsup_{\alpha\to 0} l(\alpha) < 1$, so we can choose $\epsilon > 0$ and $r \in [0,1)$:
$l(t) \leq r$ $\forall t \in [0,\epsilon)$.
Now, for $t \in [0,\infty)$, we define $h : [0,\infty) \to (0,1)$ by $h(t) = \frac{l(t)+1}{2}$.
Let $\{x_n\}_{n=1}^{\infty} \in X : x_{n+1} \in f(x_n)$ and$\forall n \in \mathbb{N}$:
$d(x_{n+1}, x_{n+2}) \leq h(\ d(x_{n+1}, x_{n+2})).\ d(x_{n+1}, x_{n+2})$
Typically, $\forall t \in [0,\infty)$, we have $h(t) < 1$ and $\{\ d(x_{n+1}, x_{n+2})\}_{n=0}^{\infty}$ is non-increasing.
So, $\{\ d(x_{n+1}, x_{n+2})\}_{n=0}^{\infty}$ converges to some point $\beta \in [0,\infty)$.
Since $\limsup_{\alpha\to\beta+0} \beta(\alpha) < 1$ and $h(\beta) < 1$, $\exists p \in [0,1)$ and $\delta > 0$:
$\forall \alpha \in [\beta, \beta+\delta]$ we have $h(\alpha) \leq p$.
$d(x_\nu, x_{\nu+1}) \leq \beta + \delta$ for some $\nu \leq n \in \mathbb{N}$
$d(x_{n+1}, x_{n+2}) \leq (h(d(x_n, x_{n+1}).d(x_n, x_{n+1})) \leq pd(x_n, x_{n+1})$
Then, $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$ and so $d(x_n, x_{n+1}) \leq d(x_n, f(x_n)) < \epsilon$. □

Definition 2. *(i) The function $\mu : [0,\infty) \to [0,\infty)$ is called auxiliary distance.*
(ii) If μ is a non-decreasing auxiliary distance function such that
$\lim_{n\to\infty} \mu^n(t) = 0$ $\forall t \in [0,\infty)$, then μ is a comparison [9] if it is continuous at $t = 0$, and $\mu(t) < t$ $\forall t > 0$.
(iii) If $r \in [1,\infty)$ and there exist positive integers k_0,$s \in (0,1)$ and a convergent series $\sum_{k=1}^{\infty} u_k$ with $u_k \geq 0$:
$r^{k+1}\mu^{k+1}(t) \leq sr^k\mu^k(t) + u_k$ $\forall k_0 \leq k$, then $\forall t \in [0,\infty)$, the monotonic auxiliary distance function μ is called $b-$comparison [10].
We denote the set of all $b-$comparison functions by B.

Lemma 4. *If μ is a $b-$comparison function, then $\sum_{k=0}^{\infty} r^k\mu^k(t)$ is convergent, increasing and continuous at $t = 0$ $\forall t \in [0,\infty)$ [9].*

Remark 2. *Each $b-$comparison function is comparison [11].*

Definition 3. *(i) The function $f : X \to X$ is $\beta-$orbital admissible where*
$\beta : X \times X \to [0, \infty)$ *if* $\forall u \in X$
$1 \leq \beta(u, f(u)) \Rightarrow 1 \leq \beta(f(u), f^2(u))$ ......(8)
(ii) If $\forall u, v \in X$, we have $1 \leq \beta(u, v)$ and
$1 \leq \beta(v, f(v)) \Rightarrow 1 \leq \beta(v, f(v))$ ......(9)
(iii) If (8) and (9) are fulfilled, then f is called triangular $\beta-$orbital addmissible [12].

Lemma 5. *If $f : (X, d, a) \to (X, d, a)$ is a triangular $\beta-$orbital admissible function and $\exists v_0 \in X$: $1 \leq \beta(v_0, f(v_0))$, then $1 \leq \beta(v_n, v_m)$ and $f(v_n) = v_{n+1}$ $\forall n, m \in \mathbb{N}$ [12]*

Теорема 3. *In the context of a whole metric space, every $\sum -$contraction permits a distinct fixed point [13].*

3. $wt-$Distance over $b-$Metric Space

Definition 4. *The metric $d : X \times X \to [0, \infty)$ is a $wt-$distance over (X, d, a) if the following hold [11]:*

(i) $\forall u, v, \omega \in X$, $a-$weighted triangle inequality $d(v, \omega) \leq a[d(u, v) + d(v, \omega)]$ holds

(ii) If $v_n \to v$ in X and $d(v, .) : X \to [0, \infty)$ such that $d(u, v) \leq \liminf_{n\to\infty} ad(u, v_n)$ $\forall v \in X$, then d is $a-$lower semicontinuous.

(iii) $\forall \epsilon > 0$, $\exists \delta > 0$: if $d(u, v) \leq \delta$ and $d(v, \omega) \leq \delta$, then $d(u, v) \leq \epsilon$.

Lemma 6. *If $p : X \times X \to [0, \infty)$ be a $wt-$distance over (X, d, a) and the sequences (a_n) , (b_n) in X and (u_n) , (v_n) in $[0, \infty)$ converging to 0,then [11]:*
i) d is a $wt-$distance over (X, d, a).
ii) if $p(a_n, b_n) \leq kn$ and $p(b_n, c) \leq un$, $\forall n \in \mathbb{N}$, then $a = c$.
iii) if $p(a_n, b_n) \leq kn$ and $p(b_n, c) \leq un$, $\forall n \in \mathbb{N}$, then (b_n) converges to c.

Теорема 4. *Let p be a $wt-$distance over (X, d, a) and $f : X \to X$, then:*
i) f is continuous.
ii) f is triangular $\beta-$orbital admissible.
ii) $\exists \alpha_0 \in X$ such that $1 \leq \beta(\alpha_0, f(\alpha_0)$.
iii) $\forall u \in X$, $1 \leq \beta(u, f(u))$
iv) $\forall u \in X$ with $1 \leq \beta(u, f(u))$ such that $u \neq f(u)$, we have
$inf\{p(u, v) + p(u, f(v))\} > 0$, then f has a fixed point.

Доказательство. i) If $a_0 \in X$ and a sequence $\{a_n\}_{n=1}^{\infty}$ is given by $a_n = f^n(a_0)$,
then $\exists b_0 \in \mathbb{N} : f(a_{b_0}) = a_{b_0} + 1$.Hence, a_{b_0} is a fixed point of the function f. □

Corollary 2. *If conditions of 3.3 hold and for $r, s \in Fix(f)$ we have $1 \leq \beta(r, s)$, then $r = s$.*

Доказательство. Let $r, s \in Fix(f)$ such that $r \neq s$, then by 3.3 we have

$$\begin{aligned} p(r, s) &\leq \beta(r, s)p(f(r), f(s)) \\ &\leq \tfrac{1}{a}\mu(p(r, s) \\ &< \mu(p(r, s)) \\ &< p(r, s) \end{aligned}$$

which is a contradiction. Therefore, f has a unique fixed point. □

Corollary 3. *If conditions of 3.3 hold and $p : X \times X \to [0, \infty)$ is a $wt-$distance on $(X^*, d, a), \forall u, v \in X$, $\forall \epsilon > 0$, $\exists \delta > 0$ such that $\epsilon < \mu(d(u, v)) < \epsilon + \delta$ implies*
$0 \leq \sigma(\beta(u, v))p(f(u), f(v), \epsilon)$ for some $\sigma \in \Sigma$ and $\mu \in \beta$ where $\mu(t) < \frac{t}{a}$ $\forall t > 0$, then a function f has a fixed point.

Доказательство. Let $\{v_n\}_{n=1}^{\infty}$ be a sequence defined as $v_n = f^n(v_0)\ \forall n \in \mathbb{N}$.

If $v_n = v_{n-1}$ and because f is a triangular $\beta-$orbital admissible, we have

$1 \le \beta(v_{n-1}, v_n)$.

$0 \le \sigma\left(\beta(u,v)p(f(u), f(v)), \epsilon\right)$

$< \epsilon - \beta(u,v)p(f(u), f(v))$

$< \mu(d(u,v) - \beta(u,v)p(f(u), f(v), f(v))$

Then $\forall u \ne v$ we get,

$\beta(u,v)p(f(u), f(v)) < \mu\left(p(u,v)\right) < p(u,v)$......(10)

Considering $u = v_{n-1}$ and $v = v_n$,

Hence, (10) holds and $p(u,v)$is a decreasing sequence that converges to the positive real number l. □

Corollary 4. *If conditions of 3.4 hold and $r, s \in Fix(f)$ such that*

i) $1 \le \beta(r,s)$

ii) $\forall \epsilon > 0\ \exists \delta > 0$ such that $\epsilon \le \mu(d(r,s)) < \epsilon + \delta$, then, $r = s$

Доказательство. Suppose that $r \ne s$ in $Fix(f)$ such that $1 \le \beta(r,s)$

By (ii) and (10) we get a contradiction, hence f has a unique fixed point. □

Теорема 5 (15). *If $(X, \le)$ is a partially ordered set such that $\forall (x,y)$ and $(z,t) \in X \times X$, $\exists$ $(a,b) \in X \times X$ such that*

$a \le x$, z and $b \le y$, t, and if (X,p) is a complete partial metric space,

$g : X \times X \to X$ is a function with the mixed monotone property on X .

Assuming that for some $\sigma \in \Sigma$, $\psi \in \Psi$ we have

$\sigma(p(g(x,y), g(a,b)) \le \sigma(\alpha p(x,a) + \beta p(y,b)) - \psi(\alpha p(x,a) + \beta p(y,a))$

$\forall \alpha + \beta < 1$, if $\exists x_0, y_0 \in X$ such that $x_0 \le g(x_0, y_0)$ and $g(y_0, x_0) \le y_0$

then, $\exists x, y \in X$ such that g has a coupled fixed point, that is; $g(x,y) = x$ and $g(y,x) = y$.

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