ЧЕБЫШЕВСКИЙ СБОРНИК
Том 24. Выпуск 3.

УДК 517 DOI 10.22405/2226-8383-2023-24-3-212-227

Комплекснозначный подход к системе нелинейных краевых задач второго порядка и многозначное отображение методом фиксированной точки

С. Р. Шиндэ

Шиндэ Сопан Раосахеб — Отделение математики, Университет Сандип (г. Нашик, Индия).
scholarswapnil@gmail.com

Аннотация

Основной целью этой рукописи является работа над прикладной частью CVMS. В этой работе мы продемонстрировали некоторые общие фиксированные результаты, а затем мы имеем дело в основном с двумя частями приложений, частью (I) комплекснозначной версии существования и общим решением нелинейной краевой задачи второго порядка с использованием функции Грина,

$$\begin{cases} \mu''(x) = Im(x, \mu(x), \mu'(x)), & \text{when } x \in [0, \mathsf{T}], \mathsf{T} > 0 \\ \mu(x_1) = \mu_1, & \\ \mu(x_2) = \mu_2, & \text{when } x_1, x_2 \in [0, \mathsf{T}]. \end{cases}$$

часть (II) Применение результатов с фиксированной точкой для многозначного отображения при настройке виртуальных машин без использования понятия непрерывности. В конце концов, для подтверждения нашего основного результата представлено несколько эквивалентных результатов и примеров.

Ключевые слова: Комплекснозначное метрическое пространство (CVMS), Общая неподвижная точка, краевая задача, последовательность Коши, многозначное отображение, свойство g.l.b., условие сжатия и полнота.

Библиография: 18 названий.

Для цитирования:
С. Р. Шиндэ. Комплекснозначный подход к системе нелинейных краевых задач второго порядка и многозначное отображение методом фиксированной точки// Чебышевский сборник, 2023, т. 24, вып. 3, с. 212–227.

CHEBYSHEVSKII SBORNIK
Vol. 24. No. 3.

UDC 517 DOI 10.22405/2226-8383-2023-24-3-212-227

Complex valued approach to the system of non-linear second order Boundary value problem and multivalued mapping via fixed point method

S. R. Shinde

Sopan Raosaheb Shinde — Department of Mathematics, Sandip University (Nashik, India).
scholarswapnil@gmail.com

Abstract

The main aim of this manuscript is to work on the application part of CVMS. In this work we have demonstrated some common fixed results and then we deal primarily with two parts of applications,
part(I) Complex valued version of existence and common solution for second order nonlinear boundary value problem using greens function,

$$\begin{cases} \mu''(x) = Im(x, \mu(x), \mu'(x)), & \text{when } x \in [0, \mathsf{T}], \mathsf{T} > 0 \\ \mu(x_1) = \mu_1, & \\ \mu(x_2) = \mu_2, & \text{when } x_1, x_2 \in [0, \mathsf{T}]. \end{cases}$$

part(II) Application of fixed point results for multivalued mapping in setting of CVMS without using notion of continuity. Eventually several equivalent results and examples are presented to sustain our Main result.

Keywords: Complex valued metric space(CVMS), Common fixed point, Boundary value Problem, Cauchy sequence, Multivalued Mapping, g.l.b. property, Contractive condition and completeness.

Bibliography: 18 titles.

For citation:
S. R. Shinde, 2023, "Complex valued approach to the system of non-linear second order Boundary value problem and multivalued mapping via fixed point method" *Chebyshevskii sbornik*, vol. 24, no. 3, pp. 212–227.

1. Introduction

Banach contraction mapping theorem [1] is a prominent tool for solving problem in nonlinear analysis, Calculus, Fuzzy Theory and so on. This principle used to establish existence and uniqueness of common solution for nonlinear integral equation and several other fields, many authors generalized this theorem [2, 3, 4] in different metric spaces within that In 2011 Azam, Khan and Fisher present the notion of complex valued metric space [5] and given some result for pair of mapping which is contraction condition satisfying a rational expression. After this establishment Rouzkard and Imdad [6] generalized some common fixed point theorems involving certain rational expressions. In 2013 Ahmad, Klin eam and Azam [7] studied multivalued mapping under generalized contractive condition subsequently Azam, Al Rawashden proved a common fixed point for multivalued mapping. Ahmad, Klin-eam, Azam [7] & Azam,Ahmad,Kumam [8] defined generalized Housdorff metric function in the setting of CVMS and established common fixed point result for multivalued mapping. Das and gupta [9] generalized banach contraction principle for rational type contractive inequality in fixed point metric space. Afterwards several researchers generalized and extended the aforesaid work with the help of different rational contraction under self and multivalued mapping in the context of CVMS for instance [7, 10, 11, 12], in addition Sinthunavarat et.al. [13, 14] studied a common solution to the Urysohn integral equation under CVMS. In this paper we studied results from [13, 14] and so on, afterward we generalized Result 10 from literature given by Azam, Jamshaid Ahmad, Klin-Eam [15] as following,

Теорема 1. *[15] Consider the complete complex valued metric space $(\Theta, \eth)$ and the function $\zeta, \xi : \Theta \to CB(\Theta)$ be multivalued mappings having global property such that,*

$$[\eta_1.\eth(\mu,\nu) + \frac{\eta_2.\eth(\nu,\xi\nu).\eth(\mu,\zeta\mu) + \eta_3\eth(\nu,\zeta\mu).\eth(\mu,\xi\nu)}{1+\eth(\mu,\nu)}] \in \omega(\zeta\mu, \xi\nu)$$

for every μ,$\nu \in \Theta$ and η_1, η_2, η_3 are non negative real number with $\eta_1 + \eta_2 + \eta_3 < 1$ Then ζ, ξ have a common fixed point.

With inspiring all above results form literature, we prove common fixed point solution to the multivalued mapping and second order nonlinear boundary value problem. In our first section we go through some important results and definition from literature.

2. Preliminaries

Definition 1. *[5] Suppose a partial order $\precsim$ defined on a complex number($\mathbb{C}$) as,*

$$\mu \precsim \nu$$

iff Real part of $(\mu) \leq$ Real part of (ν) ; Imaginary part of $(\mu) \leq$ Imaginary part of (ν). It follows,

$$\mu \leq \nu$$

- *Real part of (μ)<Real part of (ν) ; Imaginary part of $(\mu) <$ Imaginary part of (ν).*
- *Real part of $(\mu) =$ Real part of (ν) ; Imaginary part of $(\mu) =$ Imaginary part of (ν).*
- *Real part of $(\mu) <$ Real part of (ν) ; Imaginary part of $(\mu) =$ Imaginary part of (ν).*
- *Real part of $(\mu) =$ Real part of (ν) ; Imaginary part of $(\mu) <$ Imaginary part of (ν).*

Definition 2. *[5] Let Θ be non empty set & assume that the self-mapping $\eth : \Theta \to \Theta$ said to be complex valued metric if $\eth$ satisfy following condition,*

1. $0 \precsim \eth(\mu,\nu)$ *every $\mu, \nu \in \Theta$ and $\eth(\mu,\nu) = 0$ if and only if $\mu = \nu$*
2. $\eth(\mu,\nu) = \eth(\nu,\mu)$ *every $\mu,\nu \in \Theta$*
3. $\eth(\mu,\nu) \precsim \eth(\mu,\rho) + \eth(\rho,\nu)$ *for every $\mu,\nu,\rho \in \Theta$, Then $(\Theta,\eth)$ is called complex valued metric space.*

Замечание 1. *Suppose $\Theta = \mathbb{C}$ and the mapping $\eth : \Theta \times \Theta \to \mathbb{C}$ which has $\eth(\mu,\nu) = e^{i\alpha} \mid \mu - \nu \mid$ Where, $\alpha \in [0, \frac{\pi}{2}]$, Then $(\Theta,\eth)$ be complex vlaued metric space.*

Замечание 2. *Assume $\Theta = \mathbb{C}$ and the mapping $\eth : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ defined as following $\eth(\mu,\nu) =\mid \jmath_1 - \jmath_2 \mid + \iota \mid \imath_1 - \imath_2 \mid$ Where, $\mu = \jmath_1 + \iota\imath_1$ $\nu = \jmath_2 + \iota\imath_2$. Then $(\Theta,\eth)$ be complex vlaued metric space.*

Definition 3. *[5] Assume* $\{\mu_r\}$ *be a sequence in a complex valued metric space* $(\Theta, \eth)$ *and* $\mu \in \Theta$, *Then* μ *is a limit point of* $\{\mu_r\}$ *if every* $\epsilon \in \mathbb{C}$ *there exist* $r_0 \in \mathbb{N}$ *such that* $\eth(\{\mu_r\}, r) \prec \epsilon, \forall r \succ r_0$ *that is* $\lim r \to \infty, \mu_r = r$.

Definition 4. *[5] Assume* $\{\mu_r\}$ *be a sequence in a complex valued metric space* $(\Theta, \eth)$ *and* $\mu \in \Theta$, *Then* $\{\mu_r\}$ *is a cauchy sequence if for any* $\epsilon \in \mathbb{C}$ *there exist* $r_0 \in \mathbb{N}$ *such that* $\eth(\mu_r, \mu_{r+s}) \prec \epsilon, \forall r \succ r_0$ *and* $s \in \mathbb{N}$.

Definition 5. *[5] Assume* $\{\mu_r\}$ *be a sequence in a complex valued metric space* $(\Theta, \eth)$ *and* $\mu \in \Theta$, *Then* $(\Theta, \eth)$ *is said to be a complete complex valued metric space if every Cauchy sequence is convergent in* $(\Theta, \eth)$.

Proposition 1. *[5] Suppose* $(\Theta, \eth)$ *be a complex valued metric space. Then the sequence* $\{\mu_r\}$ *in* Θ *Converges to* μ *if and only if* $\mid \eth(\mu_r, \mu) \mid \to 0$ *as* $r \to \infty$.

Proposition 2. *[5] Suppose* $(\Theta, \eth)$ *be a complex valued metric space. Then the sequence* $\{\mu_r\}$ *in* Θ *is a cauchy sequence if and only if* $\mid \eth(\mu_r, \mu_{r+s}) \mid \to 0$ *as* $r \to \infty$ *where* $s \in \mathbb{N}$.

3. Main Results

Definition 6. *[7] Let* $(\Theta,\eth)$ *be a Complex valued metric space with distance* $\eth$ *then* $CB(\Theta)$ *represent family of all Bounded, nonempty, Closed subset of* Θ.

$$\omega(\mu, F_2) = \bigcup_{\nu \in F_2} \omega.\eth(\mu, \nu) = \bigcup_{\nu \in F_2} \{\ell \in \mathbb{C} : \eth(\mu, \nu) < \ell\},\ F_2 \in CB(\Theta)\ \&\ \mu \in \mathbb{C}$$

for $F_1, F_2 \in CB(\Theta)$, *we write*

$$\omega(F_1, F_2) = \left(\bigcap_{\mu \in F_2} \omega(\mu, F_2)\right) \bigcap \left(\bigcap_{\nu \in F_2} \omega(\mu, F_2)\right)$$

1. $\ell \in \mathbb{C}$ & suppose $F_1, F_2 \in CB(\Theta), \mu \in F_1$ if $\ell \in \omega(F_1, F_2)$ then $\ell \in \omega(\mu, F_2)$ for every $\mu \in F_1$ or $\ell \in \omega(F_1, \nu)$ $\forall \nu \in F_2$ which given by Ahmed et. el. as a notion of multivalued mapping.
2. Let $\mu, \nu \in \mathbb{C}$. If $\mu < \nu$, then $\omega(\nu) \subset \omega(\mu)$.
3. Assume $\ell \in \Theta$ and $F \in N(\Theta)$. If $\theta \in \omega(\ell, F)$, then $\ell \in F$.

Definition 7. *[7] Consider* $(\Theta,\eth)$ *be a Complex valued metric space with distance* $\eth$ *and* $CB(\Theta)$ *represent family of all Bounded, nonempty, Closed subset of* Θ. *Let* $\zeta : \Theta \to CB(\Theta)$ *be a multivalued mapping for* $\ell \in \Theta$ & $F \in CB(\Theta)$,

$$\Omega_\ell(F) = \{\eth(\ell, \mu) : \mu \in F\}$$

for $\ell, \nu \in \Theta$,

$$\Omega_\ell(\zeta_\nu) = \{\eth(\ell, \mu) : \mu \in \zeta_\nu\}.$$

Definition 8. *[7] Assume* $(\Theta,\eth)$ *be a Complex valued metric space and multivalued mapping* $\zeta : \Theta \to CB(\Theta)$ *said to have g.l.b. property on* $(\Theta,\eth)$ *if for all* $\mu, \nu \in \Theta$, $\Omega_\mu(\zeta\nu)$ *exists, and we used* $\eth(\mu, \zeta\nu)$ *by the g.l.b. of* $\Omega_\mu(\zeta\nu)$,

$$\eth(\mu, \zeta\nu) = inf\{\eth(\mu, \ell) : \ell \in \zeta_\nu\}.$$

Definition 9. *[7] Let* $(\Theta,\eth)$ *be a Complex valued metric space and multivalued mapping* $\zeta : \Theta \to 2^{\mathbb{C}}$ *is said to be bounded below if, for each* $\ell \in \Theta$ $\exists \ell \in \mathbb{C}$,

$$\ell_\mu < J \text{ for all } J \in \zeta_\ell.$$

Definition 10. *[7] Let (Θ,ð) be a Complex valued metric space. A subset $\digamma$ of Θ called bounded below if $\exists \ell \in \Theta$ such that $\ell < \mu$ for every $\mu \in \digamma$.*

In this Section, we present a new fixed point result under CVMS with suitable examples, Result & finally two folds of the application part.

Теорема 2. *Consider (Θ,ð) be a complete Complex valued metric space with distance ð and $\gamma : \varphi \times \varphi \to [1, \infty)$ be a map such that their exist a function $\zeta, \xi : \Theta \to \tau(\Theta)$ satisfying g.l.b. property, for each $\mu \in \Theta$ and $[\zeta_\mu], [\xi_\mu] \in CB(\Theta)$ there exist non negative real number $\eta_1, \eta_2, \eta_3, \eta_4$ with $\eta_1 + \eta_2 + \eta_3 + \eta_4 < 1$ and $\lambda(1 - \eta_2) = \eta_1$, $0 \le \lambda \le 1$, for every $\mu, \nu \in \Theta$, if we take $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n, \mu_m)\lambda < 1$ such that,*

$$[\eta_1.ð(\mu,\nu) + \frac{\eta_2.ð(\nu,\xi\nu).ð(\mu,\zeta\mu)}{1 + ð(\mu,\nu)} + \frac{\eta_3.ð(\nu,\xi\mu).ð(\mu,\xi\nu) + \eta_4.ð(\nu,\zeta\mu).ð(\mu,\xi\nu)}{1 + ð(\mu,\xi_\nu)}] \in \omega(\zeta\mu, \xi\nu) \quad (1)$$

Then their exist $\mu* \in \Theta$ such that $\mu* \in (\zeta_\mu*) \cap (\xi_\mu*)$

Proof. Suppose μ_0 be an arbitrary point in Θ. By assumption we easily find $\mu_1 \in (\zeta\mu_0)$ so we replace $[\mu = \mu_0, \nu = \mu_1]$ in inequality (1),

$$[\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\xi\mu_1)ð(\mu_0,\zeta\mu_0)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\xi\mu_0)ð(\mu_0,\xi\mu_1) + \eta_4 ð(\mu_1,\zeta\mu_0)ð(\mu_0,\xi\mu_1)}{1 + ð(\mu_0,\xi\mu_1)}] \in \omega(\zeta\mu_0, \xi\mu_1)$$

$$[\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\xi\mu_1)ð(\mu_0,\zeta\mu_0)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\xi\mu_0)ð(\mu_0,\xi\mu_1) + \eta_4 ð(\mu_1,\zeta\mu_0)ð(\mu_0,\xi\mu_1)}{1 + ð(\mu_0,\xi\mu_1)}] \in \bigcap_{\ell \in [\zeta\mu_0]} \omega(\ell, \xi\mu_1)$$

as we have $\mu_1 \in \zeta\mu_0$,

$$[\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\xi\mu_1)ð(\mu_0,\zeta\mu_0)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\xi\mu_0)ð(\mu_0,\xi\mu_1) + \eta_4 ð(\mu_1,\zeta\mu_0)ð(\mu_0,\xi\mu_1)}{1 + ð(\mu_0,\xi\mu_1)}] \in \omega(\mu_1, \xi\mu_1) \quad (2)$$

By definition,

$$[\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\xi\mu_1)ð(\mu_0,\zeta\mu_0)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\xi\mu_0)ð(\mu_0,\xi\mu_1) + \eta_4 ð(\mu_1,\zeta\mu_0)ð(\mu_0,\xi\mu_1)}{1 + ð(\mu_0,\xi\mu_1)}] \in \bigcup_{\rho \in [\xi\mu_1]} \omega(ð(\mu_1, \rho))$$

This gives there exist $\mu_2 \in \xi\mu_1$ such that,

$$[\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\xi\mu_1)ð(\mu_0,\zeta\mu_0)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\xi\mu_0)ð(\mu_0,\xi\mu_1) + \eta_4 ð(\mu_1,\zeta\mu_0)ð(\mu_0,\xi\mu_1)}{1 + ð(\mu_0,\xi\mu_1)}] \in \omega(ð(\mu_1, \mu_2)) \quad (3)$$

This gives,

$$ð(\mu_1,\mu_2) \le [\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\xi\mu_1)ð(\mu_0,\zeta\mu_0)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\xi\mu_0)ð(\mu_0,\xi\mu_1) + \eta_4 ð(\mu_1,\zeta\mu_0)ð(\mu_0,\xi\mu_1)}{1 + ð(\mu_0,\xi\mu_1)}] \quad (4)$$

using $W_\mu[\xi\nu]$ and $W_\mu[\zeta\nu]$ and $\zeta\mu_0 = \mu_1$ & $\xi\mu_1 = \mu_2$,

$$ð(\mu_1,\mu_2) \le [\eta_1 ð(\mu_0,\mu_1) + \frac{\eta_2 ð(\mu_1,\mu_2)ð(\mu_0,\mu_1)}{1 + ð(\mu_0,\mu_1)} + \frac{\eta_3 ð(\mu_1,\mu_1)ð(\mu_0,\mu_2)}{1 + ð(\mu_0,\mu_2)} + \frac{\eta_4 ð(\mu_1,\mu_1)ð(\mu_0,\mu_2)}{1 + ð(\mu_0,\mu_2)}]$$

$$\mid ð(\mu_1,\mu_2) \mid \le [\eta_1. \mid ð(\mu_0,\mu_1) \mid + \frac{\eta_2. \mid ð(\mu_1,\mu_2) \mid . \mid ð(\mu_0,\mu_1) \mid}{1 + \mid ð(\mu_0,\mu_1) \mid}]$$

Which gives,

$$| \eth(\mu_1, \mu_2) | \leq [\eta_1. | \eth(\mu_0, \mu_1) | + \eta_2. | \eth(\mu_1, \mu_2) | . \frac{| \eth(\mu_0, \mu_1) |}{1 + | \eth(\mu_0, \mu_1) |}]$$

That is,

$$| \eth(\mu_1, \mu_2) | \leq [\eta_1. | \eth(\mu_0, \mu_1) | + \eta_2. | \eth(\mu_1, \mu_2) |]$$

$$| \eth(\mu_1, \mu_2) | - \eta_2. | \eth(\mu_1, \mu_2) | \leq [\eta_1. | \eth(\mu_0, \mu_1) |]$$

$$| \eth(\mu_1, \mu_2) | \leq \frac{\eta_1}{(1 - \eta_2)}.[| \eth(\mu_0, \mu_1) |]$$

$$| \eth(\mu_1, \mu_2) | \leq \lambda.[| \eth(\mu_0, \mu_1) |] \tag{5}$$

Again likewise we take $\mu_2 \in \xi\mu_1$,

$$[\eta_1 \eth(\mu_2, \mu_1) + \frac{\eta_2 \eth(\mu_1, \xi\mu_1) \eth(\mu_2, \zeta\mu_2)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_3 \eth(\mu_1, \xi\mu_2) \eth(\mu_2, \xi\mu_1) + \eta_4 \eth(\mu_1, \zeta\mu_2) \eth(\mu_2, \xi\mu_1)}{1 + \eth(\mu_2, \xi\mu_1)}] \in \omega(\zeta\mu_2, \xi\mu_1) \tag{6}$$

$$[\eta_1 \eth(\mu_2, \mu_1) + \frac{\eta_2 \eth(\mu_1, \xi\mu_1) \eth(\mu_2, \zeta\mu_2)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_3 \eth(\mu_1, \xi\mu_2) \eth(\mu_2, \xi\mu_1) + \eta_4 \eth(\mu_1, \zeta\mu_2) \eth(\mu_2, \xi\mu_1)}{1 + \eth(\mu_2, \xi\mu_1)}] \in \bigcap_{\ell \in [\xi\mu_1]} \omega(\ell, \zeta\mu_2)$$

as we have $\mu_2 \in \xi\mu_1$,

$$[\eta_1 \eth(\mu_2, \mu_1) + \frac{\eta_2 \eth(\mu_1, \xi\mu_1) \eth(\mu_2, \zeta\mu_2)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_3 \eth(\mu_1, \xi\mu_2) \eth(\mu_2, \xi\mu_1) + \eta_4 \eth(\mu_1, \zeta\mu_2) \eth(\mu_2, \xi\mu_1)}{1 + \eth(\mu_2, \xi\mu_1)}] \in \omega(\mu_2, \zeta\mu_2) \tag{7}$$

By definition,

$$[\eta_1 \eth(\mu_2, \mu_1) + \frac{\eta_2 \eth(\mu_1, \xi\mu_1) \eth(\mu_2, \zeta\mu_2)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_3 \eth(\mu_1, \xi\mu_2) \eth(\mu_2, \xi\mu_1) + \eta_4 \eth(\mu_1, \zeta\mu_2) \eth(\mu_2, \xi\mu_1)}{1 + \eth(\mu_2, \xi\mu_1)}] \in \bigcup_{\rho \in [\zeta\mu_2]} \omega(\eth(\mu_2, \rho))$$

This gives there exist $\mu_3 \in \zeta\mu_2$ such that,

$$[\eta_1 \eth(\mu_2, \mu_1) + \frac{\eta_2 \eth(\mu_1, \xi\mu_1) \eth(\mu_2, \zeta\mu_2)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_3 \eth(\mu_1, \xi\mu_2) \eth(\mu_2, \xi\mu_1) + \eta_4 \eth(\mu_1, \zeta\mu_2) \eth(\mu_2, \xi\mu_1)}{1 + \eth(\mu_2, \xi\mu_1)}] \in \omega(\eth(\mu_2, \mu_3)) \tag{8}$$

This gives,

$$\eth(\mu_2, \mu_3) \leq [\eta_1.\eth(\mu_2, \mu_1) + \frac{\eta_2.\eth(\mu_1, \xi\mu_1).\eth(\mu_2, \zeta\mu_2)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_3 \eth(\mu_1, \xi\mu_2) \eth(\mu_2, \xi\mu_1) + \eta_4 \eth(\mu_1, \zeta\mu_2) \eth(\mu_2, \xi\mu_1)}{1 + \eth(\mu_2, \xi\mu_1)}] \tag{9}$$

using $W_\mu[\xi\nu]$ and $W_\mu[\zeta\nu]$ and $\zeta\mu_2 = \mu_3$ & $\xi\mu_1 = \mu_2$,

$$\eth(\mu_2, \mu_3) \leq [\eta_1 \eth(\mu_2, \mu_1) + \frac{\eta_2 \eth(\mu_1, \mu_2) \eth(\mu_2, \mu_3)}{1 + \eth(\mu_2, \mu_1)} + \frac{\eta_4 \eth(\mu_1, \mu_3) \eth(\mu_2, \mu_2)}{1 + \eth(\mu_2, \mu_2)} + \frac{\eta_4 \eth(\mu_1, \mu_3) \eth(\mu_2, \mu_2)}{1 + \eth(\mu_2, \mu_2)}]$$

$$\eth(\mu_2, \mu_3) \leq [\eta_1.\eth(\mu_2, \mu_1) + \frac{\eta_2.\eth(\mu_1, \mu_2).\eth(\mu_2, \mu_3)}{1 + \eth(\mu_2, \mu_1)}]$$

$$| \eth(\mu_2, \mu_3) | \leq [\eta_1. | \eth(\mu_2, \mu_1) | + \eta_2. | \eth(\mu_2, \mu_3) | . \frac{| \eth(\mu_2, \mu_1) |}{1 + | \eth(\mu_2, \mu_1) |}]$$

$$| \eth(\mu_2, \mu_3) | \leq [\eta_1. | \eth(\mu_2, \mu_1) | + \eta_2. | \eth(\mu_2, \mu_3) |]$$

$$| \eth(\mu_2, \mu_3) | - \eta_2. | \eth(\mu_2, \mu_3) | \leq [\eta_1. | \eth(\mu_2, \mu_1) |]$$

$$| \eth(\mu_2, \mu_3) | \leq \frac{\eta_1}{(1-\eta_2)} | \eth(\mu_2, \mu_1) |$$

$$| \eth(\mu_2, \mu_3) | \leq \lambda | \eth(\mu_2, \mu_1) |$$

and hence we write a sequence $\{\mu_n\}$ in Θ as,

$$| \eth(\mu_1, \mu_2) | \leq \lambda | \eth(\mu_0, \mu_1) |, | \eth(\mu_2, \mu_3) | \leq \lambda^2 | \eth(\mu_0, \mu_1) |, | \eth(\mu_n, \mu_{n+1}) | \leq \lambda^n | \eth(\mu_0, \mu_1) | .$$

for every $n \in \mathbb{N}$. and use triangle inequality, for every $m > n$

$$| \eth(\mu_n, \mu_m) | \preceq | \eth(\mu_n, \mu_{n+1}) | + | \eth(\mu_{n+1}, \mu_{n+2}) | + ... + | \eth(\mu_{m-1}, \mu_m) | \tag{10}$$

$$\leq [\lambda^n + \lambda^{n+1} + ... + \lambda^{m-1}]. | \eth(\mu_0, \mu_1) | .$$

$$\leq [\frac{\lambda^n}{1-\lambda}]. | \eth(\mu_0, \mu_1) | .$$

$$| \eth(\mu_n, \mu_m) | \leq [\frac{\lambda^n}{1-\lambda}]. | \eth(\mu_0, \mu_1) | \to 0, \Rightarrow m, n \to \infty$$

Hence which implies $\{\mu_n\}$ is a cauchy sequence in Θ. Since Θ complete, there exist $\mu* \in \Theta$ such that $\mu_n \to \mu*$ as $n \to \infty$ now we show that $\mu* \in \zeta\mu*$ and $\mu* \in \xi\mu*$ with the help of condition (1),

$$[\eta_1 \eth(\mu_{2n}, \mu*) + \frac{\eta_2 \eth(\mu*, \xi\mu*) \eth(\mu_{2n}, \zeta\mu_{2n})}{1 + \eth(\mu_{2n}, \mu*)} + \frac{\eta_3 \eth(\mu*, \xi\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)} + \frac{\eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)}]$$

$$\in \omega(\zeta\mu_{2n}, \xi\mu*)$$

$$[\eta_1 \eth(\mu_{2n}, \mu*) + \frac{\eta_2 \eth(\mu*, \xi\mu*) \eth(\mu_{2n}, \zeta\mu_{2n})}{1 + \eth(\mu_{2n}, \mu*)} + \frac{\eta_3 \eth(\mu*, \xi\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)} + \frac{\eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)}]$$

$$\in \bigcap_{\ell \in [\zeta\mu_{2n}]} \omega(\ell, \xi\mu*)$$

as we know, $\mu_{2n+1} \in \zeta\mu_{2n}$

$$[\eta_1 \eth(\mu_{2n}, \mu*) + \frac{\eta_2 \eth(\mu*, \xi\mu*) \eth(\mu_{2n}, \zeta\mu_{2n})}{1 + \eth(\mu_{2n}, \mu*)} + \frac{\eta_3 \eth(\mu*, \xi\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)} + \frac{\eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)}]$$

$$\in \omega(\mu_{2n+1}, \xi\mu*)$$

$$[\eta_1 \eth(\mu_{2n}, \mu*) + \frac{\eta_2 \eth(\mu*, \xi\mu*) \eth(\mu_{2n}, \zeta\mu_{2n})}{1 + \eth(\mu_{2n}, \mu*)} + \frac{\eta_3 \eth(\mu*, \xi\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)} + \frac{\eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)}]$$

$$\in \bigcup_{\rho \in [\xi\mu*]} \omega(\eth(\mu_{2n+1}, \rho))$$

This gives there exist $\mu_n \in \xi\mu*$ such that,

$$[\eta_1 \eth(\mu_{2n}, \mu*) + \frac{\eta_2 \eth(\mu*, \xi\mu*) \eth(\mu_{2n}, \zeta\mu_{2n})}{1 + \eth(\mu_{2n}, \mu*)} + \frac{\eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)} + \frac{\eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)}] \tag{11}$$

$$\in \omega(\eth(\mu_{2n+1}, \mu_n))$$

This gives,

$$\eth(\mu_{2n+1}, \mu_n) \preceq$$

$$\eta_1 \eth(\mu_{2n}, \mu*) + \frac{\eta_2 \eth(\mu*, \xi\mu*) \eth(\mu_{2n}, \zeta\mu_{2n})}{1 + \eth(\mu_{2n}, \mu*)} + \frac{\eta_3 \eth(\mu*, \xi\mu_{2n}) \eth(\mu_{2n}, \xi\mu*) + \eta_4 \eth(\mu*, \zeta\mu_{2n}) \eth(\mu_{2n}, \xi\mu*)}{1 + \eth(\mu_{2n}, \xi_\mu*)} \tag{12}$$

Hence the g.l.b. property for ξ,

$$\eth(\mu_{2n+1},\mu_n)\preceq$$
$$\eta_1\eth(\mu_{2n},\mu*)+\frac{\eta_2\eth(\mu*,\mu_n)\eth(\mu_{2n},\mu_{2n+1})}{1+\eth(\mu_{2n},\mu*)}+\frac{\eta_3\eth(\mu*,\mu_{2n+1})\eth(\mu_{2n},\mu_n)+\eta_4\eth(\mu*,\mu_{2n+1})\eth(\mu_{2n},\mu_n)}{1+\eth(\mu_{2n},\mu_n)} \tag{13}$$

by triangle inequality,

$$\eth(\mu*,\mu_n)\preceq\eth(\mu*,\mu_{2n+1})+\eth(\mu_{2n+1},\mu_n) \tag{14}$$

and by using (14),

$$\mid\eth(\mu*,\mu_n)\mid\preceq\mid\eth(\mu*,\mu_{2n+1})\mid+[\eta_1\mid\eth(\mu_{2n},\mu*)\mid+\frac{\eta_2\mid\eth(\mu*,\mu_n)\mid\mid\eth(\mu_{2n},\mu_{2n+1})\mid}{1+\mid\eth(\mu_{2n},\mu*)\mid}]+$$
$$[\frac{\eta_3\mid\eth(\mu*,\mu_{2n+1})\mid\mid\eth(\mu_{2n},\mu_n)\mid+\eta_4\mid\eth(\mu*,\mu_{2n+1})\mid\mid\eth(\mu_{2n},\mu_n)\mid}{1+\mid\eth(\mu_{2n},\mu_n)\mid}] \tag{15}$$

Letting as $n\to\infty$ then $\mid\eth(\mu*,\mu_n)\mid\to 0$, Using lemma (1) we say $\mu_n\to\mu*$. as we have $\xi\mu*$ closed, which gives $\mu*\in\xi\mu*$. similar we get $\mu*\in\zeta\mu*$. Hence we have $\mu*\in(\zeta_\mu*)\cap(\xi_\mu*)$.

Замечание 3. *Define map $\eth:\Theta\times\Theta\to\mathbb{C}$ & $\eth(\mu,\nu)=e^{i\frac{\pi}{6}}.\mid\mu-\nu\mid$ and $\Theta=[0,1]$ Then we say $(\Theta,\eth)$ is a complete complex valued metric space, assume $\zeta,\xi:\Theta\to CB(\Theta)$ with $\zeta\mu=[0,\frac{\mu}{4}]$ and $\xi\nu=[0,\frac{\nu}{4}]$ for all $\mu,\nu\in\Theta$. Now we need to follow the function, $\eta_i:\Theta\to[0,1]$ for all $i\in[1,4]$ as,*

$$\eta_1(\mu)=\tfrac{\mu+1}{3},\eta_2(\mu)=\tfrac{\mu}{20},\eta_3(\mu)=\tfrac{\mu}{10},\eta_4(\mu)=\tfrac{\mu}{50}.$$

Step (I) $\eta_1+\eta_2+\eta_3+\eta_4<1$ which means $\frac{\mu+1}{3}+\frac{\mu}{20}+\frac{\mu}{10}+\frac{\mu}{50}<1$ for all $\mu\in[0,1)$

Step (II) Now we calculate different values to the main result,

$\eth(\mu,\nu)=e^{i\frac{\pi}{6}}.\mid\mu-\nu\mid$

$\eth(\nu,\xi\nu)=e^{i\frac{\pi}{6}}.\mid\nu-\frac{\nu}{4}\mid$

$\eth(\mu,\xi\nu)=e^{i\frac{\pi}{6}}.\mid\mu-\frac{\nu}{4}\mid$

$\omega(\zeta\mu,\xi\nu)=\omega.(e^{i\frac{\pi}{6}}.\mid\frac{\mu}{4}-\frac{\nu}{4}\mid)$

$\eth(\mu,\zeta\mu)=e^{i\frac{\pi}{6}}.\mid\mu-\frac{\mu}{4}\mid$

$\eth(\nu,\zeta\mu)=e^{i\frac{\pi}{6}}.\mid\nu-\frac{\mu}{4}\mid$

$\eth(\mu,\zeta\nu)=e^{i\frac{\pi}{6}}.\mid\mu-\frac{\nu}{4}\mid$

By main result,

$$[\eta_1\mid\eth(\mu,\nu)\mid+\frac{\eta_2\mid\eth(\nu,\xi\nu)\mid\mid\eth(\mu,\zeta\mu)\mid}{1+\mid\eth(\mu,\nu)\mid}+\frac{\eta_3\mid\eth(\nu,\xi\mu)\mid\mid\eth(\mu,\xi\nu)\mid+\eta_4\mid\eth(\nu,\zeta\mu)\mid\mid\eth(\mu,\xi\nu)\mid}{1+\mid\eth(\mu,\xi_\nu)\mid}]$$

as we have consider,

$$\eta_1(\mu)=\tfrac{\mu+1}{3},\eta_2(\mu)=\tfrac{\mu}{20},\eta_3(\mu)=\tfrac{\mu}{10},\eta_4(\mu)=\tfrac{\mu}{50},\forall\mu,\nu\in[0,1]$$

$$\mid\frac{\mu}{4}-\frac{\nu}{4}\mid\leq[(\frac{\mu+1}{3})\mid\mu-\nu\mid+\frac{(\frac{\mu}{20})\mid\nu-\frac{\nu}{4}\mid\mid\mu-\frac{\mu}{4}\mid}{1+\mid\eth(\mu,\nu)\mid}+\frac{(\frac{\mu}{10})\mid\nu-\frac{\mu}{4}\mid\mid\mu-\frac{\nu}{4}\mid+(\frac{\mu}{50})\mid\nu-\frac{\mu}{4}\mid\mid\mu-\frac{\nu}{4}\mid}{1+\mid\mu-\frac{\nu}{4}\mid}] \tag{16}$$

It's easier to get the following step,

$$\mid\tfrac{\mu}{4}-\tfrac{\nu}{4}\mid\leq(\tfrac{\mu+1}{3}).\mid\mu-\nu\mid,\forall\mu\in\Theta$$

as we observe (27) we get, every term from right hand side non negative $\forall \mu \in \Theta$,

$$[\eta_1.\eth(\mu,\nu) + \frac{\eta_2.\eth(\nu,\xi\nu).\eth(\mu,\zeta\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_3.\eth(\nu,\xi\mu).\eth(\mu,\xi\nu) + \eta_4.\eth(\nu,\zeta\mu).\eth(\mu,\xi\nu)}{1+\eth(\mu,\xi_\nu)}] \in \omega(\zeta\mu,\xi\nu)$$

all conditions of the main theorem satisfied. Hence ζ, ξ *admits* $\mu = 0$ *as a common fixed point.*

If we consider single valued function that is, $\zeta = \xi$ in 2 we get the following result,

Proposition 3. *Consider* $(\Theta,\eth)$ *be a complete Complex valued metric space with distance* $\eth$ *and* $\gamma : \varphi \times \varphi \to [1,\infty)$ *be a map such that their exist a function* $\zeta : \Theta \to \tau(\Theta)$ *satisfying g.l.b. property, for each* $\mu \in \Theta$ *and* $[\zeta_\mu] \in CB(\Theta)$ *there exist non negative real number* $\eta_1,\eta_2,\eta_3,\eta_4$ *with* $\eta_1+\eta_2+\eta_3+\eta_4 < 1$ *and* $\lambda(1-\eta_2) = \eta_1$, $0 \le \lambda \le 1$, *for every* $\mu,\nu \in \Theta$, *if we take* $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n,\mu_m)\lambda < 1$ *such that,*

$$[\eta_1.\eth(\mu,\nu) + \frac{\eta_2.\eth(\nu,\zeta\nu).\eth(\mu,\zeta\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_3.\eth(\nu,\zeta\mu).\eth(\mu,\zeta\nu) + \eta_4.\eth(\nu,\zeta\mu).\eth(\mu,\zeta\nu)}{1+\eth(\mu,\zeta_\nu)}] \in \omega(\zeta\mu,\zeta\nu) \quad (17)$$

Then their exist $\mu* \in \Theta$ *such that* $\mu* \in (\zeta_\mu *)$

By using $\eta_4 = 0$ in main result 2 we come under following Result,

Proposition 4. *Suppose* $(\Theta,\eth)$ *be a complete Complex valued metric space with distance* $\eth$ *and* $\gamma : \varphi \times \varphi \to [1,\infty)$ *be a map such that their exist a function* $\zeta,\xi : \Theta \to \tau(\Theta)$ *satisfying g.l.b. property, for each* $\mu \in \Theta$ *and* $[\zeta_\mu],[\xi_\mu] \in CB(\Theta)$ *there exist non negative real number* $\eta_1,\eta_2,\eta_3,\eta_4$ *with* $\eta_1+\eta_2+\eta_3 < 1$ *and* $\lambda(1-\eta_2) = \eta_1$, $0 \le \lambda \le 1$, *for every* $\mu,\nu \in \Theta$, *if we take* $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n,\mu_m)\lambda < 1$ *such that,*

$$[\eta_1.\eth(\mu,\nu) + \frac{\eta_2.\eth(\nu,\xi\nu).\eth(\mu,\zeta\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_3.\eth(\nu,\xi\mu).\eth(\mu,\xi\nu)}{1+\eth(\mu,\xi_\nu)}] \in \omega(\zeta\mu,\xi\nu) \quad (18)$$

Then their exist $\mu* \in \Theta$ *such that* $\mu* \in (\zeta_\mu *) \cap (\xi_\mu *)$

By using $\eta_3 = 0$ in main result 2 we come under following Result,

Proposition 5. *Consider* $(\Theta,\eth)$ *be a complete Complex valued metric space with distance* $\eth$ *and* $\gamma : \varphi \times \varphi \to [1,\infty)$ *be a map such that their exist a function* $\zeta,\xi : \Theta \to \tau(\Theta)$ *satisfying g.l.b. property, for each* $\mu \in \Theta$ *and* $[\zeta_\mu],[\xi_\mu] \in CB(\Theta)$ *there exist non negative real number* η_1,η_2,η_4 *with* $\eta_1+\eta_2+\eta_4 < 1$ *and* $\lambda(1-\eta_2) = \eta_1$, $0 \le \lambda \le 1$, *for every* $\mu,\nu \in \Theta$, *if we take* $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n,\mu_m)\lambda < 1$ *such that,*

$$[\eta_1.\eth(\mu,\nu) + \frac{\eta_2.\eth(\nu,\xi\nu).\eth(\mu,\zeta\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_4.\eth(\nu,\zeta\mu).\eth(\mu,\xi\nu)}{1+\eth(\mu,\xi_\nu)}] \in \omega(\zeta\mu,\xi\nu) \quad (19)$$

Then their exist $\mu* \in \Theta$ *such that* $\mu* \in (\zeta_\mu *) \cap (\xi_\mu *)$

Замечание 6. *Let* $(\Theta,\eth)$ *be a complete Complex valued metric space with distance* $\eth$. *If we use* $\mathbb{C} = \mathbb{R}$, *then we get* $(\Theta,\eth)$ *be a metric space with distance* $\eth$. *Moreover for* $\mathbb{A},\mathbb{B} \in CB(\Theta)$, $\mathbb{H}(\mathbb{A},\mathbb{B}) = inf(\mathbb{A},\mathbb{B})$ *known as Hausdorff distance induced by* $\eth$.

Замечание 7. *If we put* $\eta_3 = \eta_4 = 0$ *in main theorem* 2 *we get the Result (10) from literature given by Azam, Jamshaid Ahmad, Klin-Eam [15] .*

Замечание 8. *With the help of mapping* ζ,ξ *in Theorem* 2 *subsequently results* 3, 4, 5 & *point dependent different control function like*$\{\eta_i, 1 \le we \le 4\ \}$ *one can deduce multitude of Result from literature for multivalued mapping in CVMS including Banach contraction Result.*

4. Applications

4.1. Application Part.I

Now subsequently we demonstrating the application part of the main theorem. We use Result 5 for finding existence and common solution of Nonlinear 2^{nd} order Boundary value problem.

$$\begin{cases} \mu''(x) = Im(x, \mu(x), \mu'(x)), & \text{when } x \in [0, \mathsf{T}], \mathsf{T} > 0 \\ \mu(x_1) = \mu_1, & \\ \mu(x_2) = \mu_2, & \text{when } x_1, x_2 \in [0, \mathsf{T}]. \end{cases} \tag{20}$$

Where $Im : [0, \mathsf{T}] \times \tau(\Theta) \times \tau(\Theta) \to \tau(\Theta)$ is continuous and which equal to the integral type equation,

$$\mu(x) - \lambda(x) = \int_{x_1}^{X_2} \chi(x, \psi).Im(\psi, \mu(\psi), \mu'(\psi))d\psi, \forall x \in [0, \mathsf{T}]. \tag{21}$$

Where $\chi(x, \psi)$ is a greens function which is written as,

$$\chi(x, \psi) = \begin{cases} \frac{(x_2 - x)(\psi - x_1)}{(x_2 - x_1)}, & \text{when } x_1 \leq \psi \leq x \leq x_2 \\ \\ \frac{(x_2 - \psi)(x - x_1)}{(x_2 - x_1)}, & \text{when } x_1 \leq x \leq \psi \leq x_2 \end{cases}$$

$\gamma(x)$ having $\gamma'' = 0$, $\gamma(x_1) = \mu_1$, $\gamma(x_2) = \mu_2$ and greens function $\chi(x, \psi)$ satisfies following property,

$$\int_{x_1}^{X_2} \chi(x, \psi)d\psi \leq \frac{(x_2 - x_1)^2}{8}$$

$$\int_{x_1}^{X_2} \chi_x(x, \psi)d\psi \leq \frac{(x_2 - x_1)}{2}$$

Throught application part , we use

$$\Delta_i(\mu(x)) = \int_{x_1}^{X_2} \chi(x, \psi).Im_i(\psi, \mu(\psi), \mu'(\psi))d\psi,$$

$$\Delta'_i(\mu(x)) = \int_{x_1}^{X_2} \chi(x, \psi).Im_i(\psi, \mu(\psi), \mu'(\psi))d\psi, \forall x \in [0, \mathsf{T}]. \tag{22}$$

we used [16, 17, 18] for more detailed understanding and defined the operator to prove existence & uniqueness solution for integral,

$$\digamma_1(\mu)(x) - \gamma(x) = \int_{x_1}^{X_2} \chi(x, \psi).Im_1(\psi, \mu(\psi), \mu'(\psi))d\psi, \forall x \in [0, \mathsf{T}].$$

$$\digamma_2(\mu)(x) - \gamma(x) = \int_{x_1}^{X_2} \chi(x, \psi).Im_2(\psi, \mu(\psi), \mu'(\psi))d\psi, \forall x \in [0, \mathsf{T}]. \tag{23}$$

Where $Im_1, Im_2 \in ([0, \mathsf{T}] \times \tau(\Theta) \times \tau(\Theta), \tau(\Theta)\ )$, $\mu \in C'([0, \mathsf{T}], \tau(\Theta))$ & $\gamma \in C([0, \mathsf{T}], \tau(\Theta))$

Теорема 3. *Consider integral equation 22, we assume following hypothesis satisfies for every $x \in [0, \mathsf{T}]$:*

1. $Im_1, Im_2 : [0, \mathsf{T}] \times \tau(\Theta) \times \tau(\Theta) \to \tau(\Theta)$ *are nondecreasing to their* 2^{nd} *and* 3^{rd} *variables;*

2.

$$\mu_0(x) - \gamma(x) \leq \int_{x_1}^{X_2} \chi(x,\psi).Im_i(\psi,\mu(\psi),\mu'(\psi))d\psi, \forall x_1, x_2 \in [0,\mathsf{T}].$$

and $\exists\ \eta_1, \eta_2 : \mathbb{C} \to [0,1)$ *has*
(a) $\varsigma, \vartheta > 0$ *and* $x_1, x_2 \in [0,\mathsf{T}]$,

$$\vartheta.\frac{(x_2-x_1)^2}{8} + \varsigma.\frac{(x_2-x_1)}{2} < 1 \tag{24}$$

(b) for all $\mu, \nu \in \Theta$ & $x \in [0,\mathsf{T}]$,

$$\mid Im_1(x,\mu(x),\mu'(x)) - Im_2(x,\nu(x),\nu'(x)) \mid \preceq \eta_1(\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x)) + \eta_2(\max_{x\in[x_1,x_2]} \mathfrak{C}_{\mu\nu}(x)) + \eta_4(\max_{x\in[x_1,x_2]} \mathfrak{D}_{\mu\nu}(x)) \tag{25}$$

(c) $\eta_1 + \eta_2 + \eta_4 < 1$ *and mapping* $\lambda : \mathbb{C}_+ \to [0,1)$ *defined as,*

$$\lambda(\mu) = \frac{\eta_1(\mu)}{1-\eta_2(\mu)}, \forall \mu \in \mathbb{C}_+ \tag{26}$$

Where,

$$\mathfrak{B}_{\mu\nu}(x) = \vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid .\sqrt{1+a^2}e^{itana}$$

$$\mathfrak{C}_{\mu\nu}(x) = \frac{\vartheta \mid \Delta_2\nu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_2'\nu(x) - \Delta_2'\nu(x) \mid}{1 + (\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x))} \times \vartheta \mid \Delta_2\nu(x) - \Delta_2\nu(x) \mid$$

$$+ \varsigma \mid \Delta_2'\nu(x) - \Delta_2'\nu(x) \mid \sqrt{1+a^2}e^{itana}$$

$$\mathfrak{D}_{\mu\nu}(x) = \frac{\vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid}{1 + \vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid} \times \vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid$$

$$+ \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid \sqrt{1+a^2}e^{itana}$$

then the nonlinear integral system,

$$\mu(x) - \gamma(x) = \int_{x_1}^{X_2} \chi(x,\psi).Im_i(\psi, x_1(\psi), x_1'(\psi))d\psi, \forall x \in [0,\mathsf{T}], i \in \{1,2\} \tag{27}$$

admits a common solution in $C'([x_1,x_2],\tau(\Theta))$.

Proof. Let $\beth = C_1'([x_1,x_2],\tau(\Theta))$ and $\eth : \beth \times \beth \to \mathbb{C}$ defined as,

$$\eth(\nu,\mu) = \max_{x\in[x_1,x_2]} (\vartheta \mid \nu(x) - \mu(x) \mid) + (\vartheta \mid \nu'(x) - \mu'(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha}$$

Now we define operators for integral, $F_i : \beth \to \beth, i = 1,2$ as

$$F_i(\mu)(x) - \gamma(x) = \int_{x_1}^{X_2} \chi(x,\psi).Im_i(\psi, x(\psi), x'(\psi))d\psi, \forall x \in [0,\mathsf{T}], i \in \{1,2\}$$

where as, $Im_1, Im_2 \in C([0,\mathsf{T}] \times \tau(\Theta) \times \tau(\Theta), \tau(\Theta)\ )$, $\mu \in C'([0,\mathsf{T}],\tau(\Theta))$ & $\gamma \in C([0,\mathsf{T}],\tau(\Theta))$ for all $\mu,\nu \in \Theta$ we get,

$$F_1(\mu) = \{\mu_1 \in [x_1,x_2], \mu_1(x) = \int_{x_1}^{X_2} \chi(x,\psi)Im_i(\psi,\mu(\psi),\mu'(\psi))d\psi, \forall x \in [0,\mathsf{T}], F_i(\mu)(\mu_1) \geq \alpha\}, i = 1,2$$

$$\begin{cases} \eth(\mu,\nu) = \max_{x\in[x_1,x_2]}(\vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha} \\ \eth(F_1\mu,\nu) = \max_{x\in[x_1,x_2]}(\vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha} \\ \eth(F_2\mu,\nu) = \max_{x\in[x_1,x_2]}(\vartheta \mid \Delta_2\nu(x) - \Delta_1\mu(x) \mid + \varsigma \mid \Delta_2'\nu(x) - \Delta_1'\mu(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha} \\ \eth(\mu,F_1\nu) = \max_{x\in[x_1,x_2]}(\vartheta \mid \Delta_1\mu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_2'\nu(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha} \\ \eth(\nu,F_2\nu) = \max_{x\in[x_1,x_2]}(\vartheta \mid \Delta_2\nu(x) - \Delta_2\nu(x) \mid + \varsigma \mid \Delta_2'\nu(x) - \Delta_2'\nu(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha} \\ \eth(\mu,F_1\mu) = \max_{x\in[x_1,x_2]}(\vartheta \mid \Delta_1\mu(x) - \Delta_1\mu(x) \mid + \varsigma \mid \Delta_1'\mu(x) - \Delta_1'\mu(x) \mid).\sqrt{1+\alpha^2}e^{itan\alpha} \end{cases}$$

For every $x \in [x_1, x_2]$ and with the help of (25) we get,

$$\mid F_1(\mu)(x) - F_2(\nu)(x) \mid = \mid \int_{x_1}^{X_2} \chi(x,\psi).[Im_1(\psi,\mu(\psi),\mu'(\psi)) - Im_2(\psi,\nu(\psi),\nu'(\psi))] \mid d\psi$$

$$\leq \int_{x_1}^{X_2} \mid \chi(x,\psi) \mid\mid [Im_1(\psi,\mu(\psi),\mu'(\psi)) - Im_2(\psi,\nu(\psi),\nu'(\psi))] \mid\mid d\psi$$

$$\leq \frac{(x_2-x_1)^2}{8}.\eta_1.(\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x)) + \eta_2.(\max_{x\in[x_1,x_2]} \mathfrak{C}_{\mu\nu}(x)) + \eta_4.(\max_{x\in[x_1,x_2]} \mathfrak{D}_{\mu\nu}(x)) \tag{28}$$

$$\mid (F_1(\mu))'(x) - (F_2(\nu))'(x) \mid = \mid \int_{x_1}^{X_2} \chi_x(x,\psi).[Im_1(\psi,\mu(\psi),\mu'(\psi)) - Im_2(\psi,\nu(\psi),\nu'(\psi))] \mid d\psi$$

$$\leq \int_{x_1}^{X_2} \mid \chi_x(x,\psi) \mid\mid [Im_1(\psi,\mu(\psi),\mu'(\psi)) - Im_2(\psi,\nu(\psi),\nu'(\psi))] \mid\mid d\psi$$

$$\leq \frac{x_2-x_1}{2}.\eta_1.(\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x)) + \eta_2.(\max_{x\in[x_1,x_2]} \mathfrak{C}_{\mu\nu}(x)) + \eta_4.(\max_{x\in[x_1,x_2]} \mathfrak{D}_{\mu\nu}(x)) \tag{29}$$

With the help of (29),(28) we write

$$\eth(\mid F_1(\mu), F_2(\nu)(x) \mid) \leq$$

$$(\vartheta\frac{(x_2-x_1)^2}{8} + \varsigma\frac{(x_2-x_1)}{2})\eta_1(\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x)) + \eta_2(\max_{x\in[x_1,x_2]} \mathfrak{C}_{\mu\nu}(x)) + \eta_4(\max_{x\in[x_1,x_2]} \mathfrak{D}_{\mu\nu}(x))$$

and equation (24) gives,

$$\eth(F_1(\mu), F_2(\nu)(x)) < \eta_1(\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x)) + \eta_2(\max_{x\in[x_1,x_2]} \mathfrak{C}_{\mu\nu}(x)) + \eta_4(\max_{x\in[x_1,x_2]} \mathfrak{D}_{\mu\nu}(x))$$

which mean,

$$\eta_1(\max_{x\in[x_1,x_2]} \mathfrak{B}_{\mu\nu}(x)) + \eta_2(\max_{x\in[x_1,x_2]} \mathfrak{C}_{\mu\nu}(x)) + \eta_4(\max_{x\in[x_1,x_2]} \mathfrak{D}_{\mu\nu}(x)) \in \omega(\eth(F_1(\mu), F_2(\nu)(x)))$$

$$[\eta_1\eth(\mu,\nu) + \frac{\eta_2\eth(\nu,F_2\nu)\eth(\mu,F_1\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_4\eth(\nu,F_1\mu)\eth(\mu,F_2\nu)}{1+\eth(\mu,F_2\nu)}] \in \omega(F_1\mu, F_2\nu)$$

then we write,

$$[\eta_1\eth(\mu,\nu) + \frac{\eta_2\eth(\nu,\xi\nu)\eth(\mu,\zeta\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_4\eth(\nu,\zeta\mu)\eth(\mu,\xi\nu)}{1+\eth(\mu,\xi_\nu)}] \in \omega(\zeta\mu, \xi\nu)$$

Hence, application part of Result (2.4) gives, Inside Θ F_1, F_2 admits common fixed point.

$$\mu(x) - \gamma(x) = \int_{x_1}^{X_2} \chi(x,\psi).Im_i(\psi, x_1(\psi), x_1'(\psi))d\psi, \forall x \in [0, \mathsf{T}], i \in \{1,2\}$$

admits a common solution. Hence second order nonlinear boundary value problem admits a solution in Θ.

In this second part we go through some fixed point Result for multivalued mapping as the application part for the main result.

4.2. Application Part.II

Теорема 4. *Consider $(\Theta,\eth)$ be a complete CVMS with distance $\eth$ and $\gamma : \varphi \times \varphi \to [1,\infty)$ be a map such that their exist a function $F_1, F_2 : \Theta \to \tau(\Theta)$ satisfying g.l.b. property, for each $\mu \in \Theta$ and $[F_1\mu], [F_2\mu] \in CB(\Theta)$ there exist non negative real number $\eta_1, \eta_2, \eta_3, \eta_4$ with*
(a) $\eta_1 + \eta_2 + \eta_3 + \eta_4 < 1$ and $\lambda(1-\eta_2) = \eta_1$, $0 \le \lambda \le 1$,
(b) for every $\mu, \nu \in \Theta$, if we take $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n, \mu_m)\lambda < 1$ such that,

$$[\eta_1\eth(\mu,\nu) + \frac{\eta_2\eth(\nu,F_2\nu)\eth(\mu,F_1\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_3\eth(\nu,F_2\mu)\eth(\mu,F_2\nu) + \eta_4\eth(\nu,F_1\mu)\eth(\mu,F_2\nu)}{1+\eth(\mu,F_2\nu)}] \in \omega(F_1\mu, F_2\nu) \tag{30}$$

Then their exist $\mu* \in \Theta$ such that $\mu* \in (F_1\mu*) \cap (F_2\mu*)$.

Proof. Consider mapping $\Xi_1, \Xi_2 : \Theta \to \tau(\Theta)$ defined by following,

$$\Xi_1(\mu)(t) = \begin{cases} \beta, & \text{when } t \in F_1\mu \\ 0, & \text{when } t \notin F_1\mu \end{cases}$$

$$\Xi_2(\mu)(t) = \begin{cases} \beta, & \text{when } t \in F_2\mu \\ 0, & \text{when } t \notin F_2\mu \end{cases} \tag{31}$$

Where $\beta \in (0,1]$ we get,
$\Xi_1(\mu) = \{t : \Xi_1(\mu)(t) \ge \beta\} = F_1(\mu)$ & $\Xi_2(\mu) = \{t : \Xi_2(\mu)(t) \ge \beta\} = F_2(\mu)$ Hence by theorem (2.1) we get, $\mu* \in \Theta$ such that $\mu* \in (\Xi_1(\mu*) \cap \Xi_2(\mu*)) = (F_1\mu*) \cap (F_2\mu*)$.

If we use single multivalued mapping then we get following result,

Proposition 6. *Consider $(\Theta,\eth)$ be a complete Complex valued metric space with distance $\eth$ and $\gamma : \varphi \times \varphi \to [1,\infty)$ be a map such that their exist a function $F : \Theta \to \tau(\Theta)$ satisfying g.l.b. property, for each $\mu \in \Theta$ and $F \in CB(\Theta)$ there exist non negative real number $\eta_1, \eta_2, \eta_3, \eta_4$ with*
(a) $\eta_1 + \eta_2 + \eta_3 + \eta_4 < 1$ and $\lambda(1-\eta_2) = \eta_1$, $0 \le \lambda \le 1$,
(b) for every $\mu, \nu \in \Theta$, if we take $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n, \mu_m)\lambda < 1$ such that,

$$[\eta_1.\eth(\mu,\nu) + \frac{\eta_2.\eth(\nu,F\nu).\eth(\mu,F\mu)}{1+\eth(\mu,\nu)} + \frac{\eta_3.\eth(\nu,F\mu).\eth(\mu,F\nu) + \eta_4.\eth(\nu,F\mu).\eth(\mu,F\nu)}{1+\eth(\mu,F\nu)}] \in \omega(F\mu, F\nu) \tag{32}$$

Then their exist $\mu* \in \Theta$ such that $\mu* \in (F\mu*)$.

By taking $\eta_4 = 0$ in theorem 4 we come under following result,

Proposition 7. *Consider* (Θ,$\eth$) *be a complete Complex valued metric space with distance* $\eth$ *and* $\gamma : \varphi \times \varphi \to [1, \infty)$ *be a map such that their exist a function* $F_1, F_2 : \Theta \to \tau(\Theta)$ *satisfying g.l.b. property, for each* $\mu \in \Theta$ *and* $[F_1\mu], [F_2\mu] \in CB(\Theta)$ *there exist non negative real number* η_1, η_2, η_3 *with*
(a) $\eta_1 + \eta_2 + \eta_3 < 1$ *and* $\lambda(1 - \eta_2) = \eta_1$, $0 \le \lambda \le 1$,
(b) for every $\mu, \nu \in \Theta$, *if we take* $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n, \mu_m)\lambda < 1$ *such that,*

$$[\eta_1.\eth(\mu, \nu) + \frac{\eta_2.\eth(\nu, F_2\nu).\eth(\mu, F_1\mu)}{1 + \eth(\mu, \nu)} + \frac{\eta_3.\eth(\nu, F_2\mu).\eth(\mu, F_2\nu)}{1 + \eth(\mu, F_2\nu)}] \in \omega(F_1\mu, F_2\nu) \tag{33}$$

Then their exist $\mu* \in \Theta$ *such that* $\mu* \in (F_1\mu*) \cap (F_2\mu*)$

By taking $\eta_3 = 0$ in theorem 4 we get following Result,

Proposition 8. *Consider* (Θ,$\eth$) *be a complete Complex valued metric space with distance* $\eth$ *and* $\gamma : \varphi \times \varphi \to [1, \infty)$ *be a map such that their exist a function* $F_1, F_2 : \Theta \to \tau(\Theta)$ *satisfying g.l.b. property, for each* $\mu \in \Theta$ *and* $[F_1\mu], [F_2\mu] \in CB(\Theta)$ *there exist non negative real number* η_1, η_2, η_4 *with*
(a) $\eta_1 + \eta_2 + \eta_4 < 1$ *and* $\lambda(1 - \eta_2) = \eta_1$, $0 \le \lambda \le 1$,
(b) for every $\mu, \nu \in \Theta$, *if we take* $\mu_0 \in \Theta$, $\lim_{m,n\to\infty} \gamma(\mu_n, \mu_m)\lambda < 1$ *such that,*

$$[\eta_1.\eth(\mu, \nu) + \frac{\eta_2.\eth(\nu, F_2\nu).\eth(\mu, F_1\mu)}{1 + \eth(\mu, \nu)} + \frac{\eta_4.\eth(\nu, F_1\mu).\eth(\mu, F_2\nu)}{1 + \eth(\mu, F_2\nu)}] \in \omega(F_1\mu, F_2\nu)$$

Then their exist $\mu* \in \Theta$ *such that* $\mu* \in (F_1\mu*) \cap (F_2\mu*)$.

5. Conclusion

It is proven fact the CVMS and its generalization in the setting of various topological spaces can be applied to find fixed point solution in different fields including integral type equation, differential equation, and many more. Through this work, we have established some new fixed-point results in the setting of complex valued metric space. We provided some examples, results and their assumption to sustain our main results. Subsequently in the application part we proved following two fold:

- fold (I) Complex valued version of existence and common solution for second order nonlinear boundary value problem using greens function;
- fold (II) Application of fixed point results for multivalued mapping in setting of CVMS.

СПИСОК ЦИТИРОВАННОЙ ЛИТЕРАТУРЫ

1. Банах С. Операции над ансамблями абстракций и их применение к интегральным уравнениям // *Фонд. Математика*, 3, (1922), С. 133–181.

2. Цирик Л.Б. , Обобщение принципа сжатия Банаха // *Труды Амер. Мат. Общ.*45, (1974) 267–273.

3. Досенович Т., Ракич Д., Карик Б., Раденович С. Многозначные обобщения результатов с фиксированной точкой в нечетких метрических пространствах // *Нелинейный анализ. Моделирование и Контроль*, 21, (2016), С. 211–222.

4. Шатанави У., Раджич В., Раденович С., Аль-Раващде. Теорема типа Мидзогучи—-Такахаши в метрических пространствах tvs-конуса // *приложение теории неподвижных точек*, (2012).

5. Азам, Фишер, Хан. Общие теоремы о неподвижной точке в комплекснозначных метрических пространствах // *Числ. Функция. Анал. Оптимум*, 32, (2011), С. 243–253.

6. Рузкард, Имдад. Некоторые общие теоремы о неподвижной точке в комплекснозначных метрических пространствах // *Компьютеры и математика с приложениями*, 64, (2012), С. 1866–1874.

7. Ахмад, Клин-эам, Азам. Общие фиксированные точки для многозначных отображений в комплекснозначных метрических пространствах с приложениями // *Абстрактный и прикладной анализ*, (2013), С. 1-12.

8. Азам, Ахмад, Кумам. Общие теоремы о неподвижной точке для многозначных отображений в комплекснозначных метрических пространствах // *Журнал неравенств и приложений*, № 1, (2013) С. 578.

9. Дас, Гупта. Расширение принципа банахова сокращения посредством рационального выражения // *Индиан Дж. Чистое приложение. Математика*, 6 (1975), С. 1455-1458.

10. Клин-эам, Суанум. Некоторые общие теоремы о фиксированной точке для обобщенных сжимающих отображений на комплекснозначных метрических пространствах, *Абстрактный и прикладной анализ*, (2013).

11. Кутби, Ахмад, Азам, Аль-Раващде. Обобщенные результаты с общей фиксированной точкой с помощью свойства наибольшей нижней границы, *Журнал Математика*, (2014), С. 1-11.

12. Синтунаварат, Кумам. Обобщенные общие теоремы о неподвижной точке в комплекснозначных метрических пространствах и приложениях, *Журнал неравенств и приложений*, (2012), С. 1-12.

13. Синтунаварат, Б.Зада, Сарвар. Общее решение интегральных уравнений Урисона с помощью общих результатов с фиксированной точкой в комплекснозначных метрических пространствах // *Журнал Королевской Академии точных, физических и естественных наук. Серия А. Математика*, 111 (2017), С. 531-545.

14. Синтунаварат, Чо, Кумам. Подход к интегральным уравнениям Урисона с использованием общих неподвижных точек в комплекснозначных метрических пространствах // *Достижения в области разностных уравнений*, (2013), С. 1-14.

15. Джамшаид Ахмад, Клин Эам, Азам. Общая фиксированная точка для многозначных отображений в комплекснозначном метрическом пространстве с приложением // *Абстрактный и прикладной анализ*,2013, Ид. статьи 854965, (2013), С. 12.

16. Лакшмикантам, Мохапатра. Теория нечетких дифференциальных уравнений и включений // *Тейлор и Фрэнсис*, 2003.

17. Пури, Ралеску. Нечеткие случайные величины // *Журнал. Математика. Анал. Приложение* 114 (1986), С. 409-422.

18. Нашин, Ветро, Кумам, Кумам. Теоремы о неподвижной точке для нечетких отображений и приложения к обычным нечетким дифференциальным уравнениям // *Достижения в области разностных уравнений*, (2014), С. 1-14.

REFERENCES

1. S. Banach, 1922, "Sur les operations dans les ensembles abstraits et leurs application aux equations integrales", *Fund. Math.*, 3, pp. 133-–181.
2. L.B. Ciric, 1974, "A generalization of Banach's contraction principle", *Proc. Amer.Math. Soc.*, 45, pp. 267-–273.
3. T. Dosenovic, D. Rakic, B. Caric, S. Radenovic, 2016, "Multivalued generalizations of fixed point results in fuzzy metric spaces", *Nonlinear Anal. Model. Control*, 21, pp. 211-–222.
4. W.Shatanawi,V.Rajic, S.Radenovic, Al-Rawashdeh, 2012, "Mizoguchi–Takahashi-type theorem in tvs-cone metric spaces", *Fixed Point Theory Appl.*.
5. Azam, Fisher, Khan, 2011, "Common fixed point theorems in complex valued metric spaces", *Numer. Funct. Anal. Optim.*, 32, pp. 243-–253.
6. Rouzkard, Imdad, 2012, "Some common fixed point theorems on complex valued metric spaces", *Computers & Mathematics with Applications*, 64, pp. 1866-–1874.
7. Ahmad, Klin-eam, Azam, 2013, "Common fixed points for multivalued mappings in complex valued metric spaces with applications", *Abstract and Applied Analysis*, pp. 1-–12.
8. Azam,Ahmad,Kumam, 2013, "Common fixed point theorems for multi-valued mappings in complex-valued metric spaces", *Journal of Inequalities and Applications*, № 1, pp. 578.
9. Das, Gupta, 1975, "An extension of Banach contraction principle through rational expression", *Indian J. Pure Appl. Math.*, 6, pp. 1455-–1458.
10. Klin-eam, Suanoom, 2013, "Some common fixed-point theorems for generalized contractive mappings on complex-valued metric spaces", *Abstr. Appl. Anal.*.
11. Kutbi, Ahmad, Azam, Al-Rawashdeh, 2014, "Generalized common fixed point results via greatest lower bound property", *J. Appl. Math.*, pp. 1—11.
12. Sintunavarat, Kumam, 2012, "Generalized common fixed point theorems in complex valued metric spaces and applications", *J.Inequal.Appl.*, pp. 1—12.
13. Sintunavarat, B.Zada, Sarwar, 2017, "Common solution of Urysohn integral equations with the help of common fixed point results in complex valued metric spaces", *Rev. R. Acad.Exactas F ıs. Nat. Ser. A Mat. RACSAM*, 111, pp. 531-–545.
14. Sintunavarat, Cho, Kumam, 2013, "Urysohn integral equations approach by common fixed points in complex valued metric spaces", *Adv. Difference Equ.*, pp. 1—14.
15. Jamshaid Ahmad, Klin Eam, Azam, 2013, "Common fixed point for Multivalued mappings in complex valued metric space with Application", *Abstract and Applied Analysis Vol.*, Article ID 854965, (2013), pp. 12.
16. Lakshmikantham, Mohapatra, 2003, "Theory of Fuzzy Differential Equations and Inclusions", *Taylor & Francis, Ltd.*, London.
17. Puri, Ralescu, 1986, "Fuzzy random variables", *J. Math. Anal. Appl.*, 114, pp. 409-–422.
18. Nashine, Vetro, Kumam, Kumam, 2014, "Fixed point theorems for fuzzy mappings and applications to ordinary fuzzy differential equations", *Adv. Difference Equ.*, pp. 1-–14.

Получено: 26.01.2023
Принято в печать: 12.09.2023